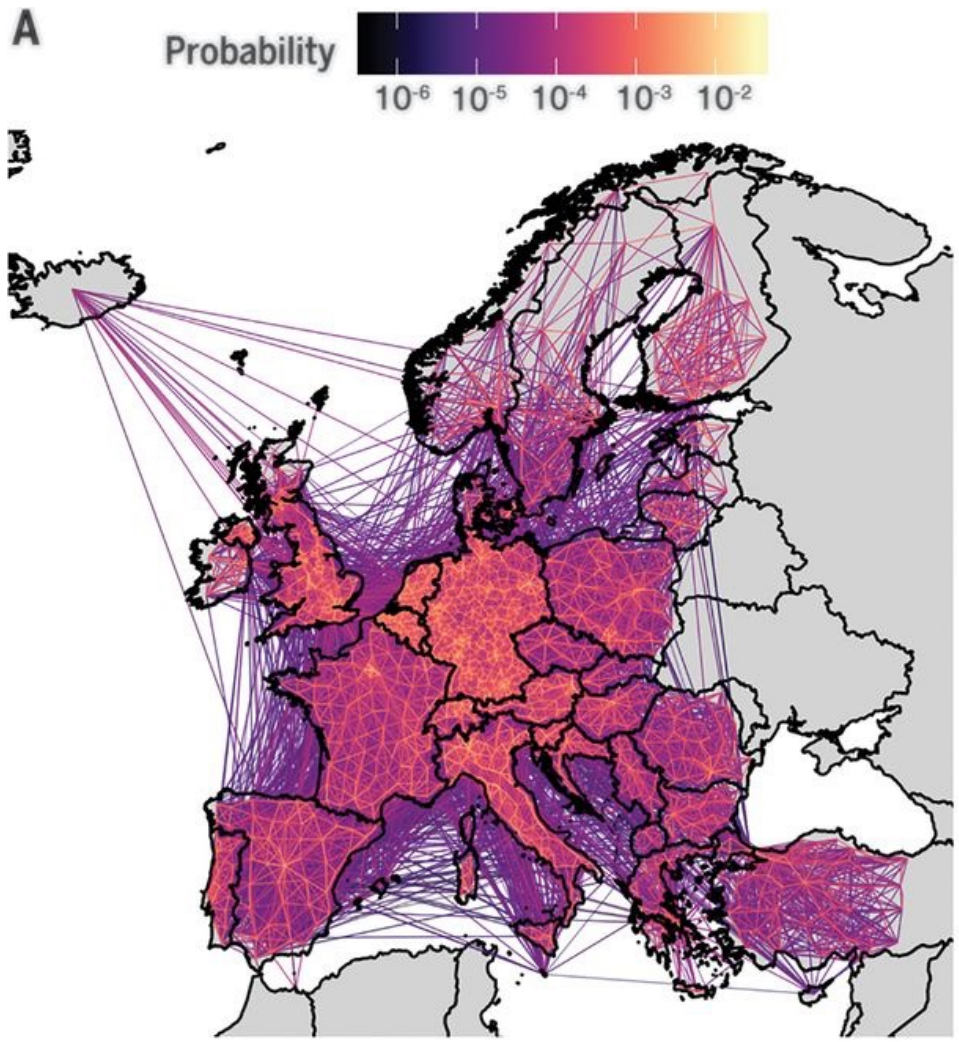
Spatial Models of Infectious Disease

EEID 2026

My details

- Dr Nick W Ruktanonchai
- nrukt00@vt.edu
- Human mobility & infectious disease
  - How does travel affect where/when infectious diseases spread?



Assumptions of simple SIR model

- Isolated, well-mixed models work well for small communities
- β: number of infectious contacts by an infectious host per day
  - Assumes the contacts are *possible*
- $\frac{I}{N}$: fraction of hosts who are infectious
  - Assumes all susceptible hosts can be contacted

$$\frac{\mathrm{d}S}{\mathrm{dt}} = -\beta S \frac{I}{N}$$

$$\frac{\mathrm{d}I}{\mathrm{dt}} = \beta S \frac{I}{N} - \gamma I$$

$$\frac{\mathrm{d}R}{\mathrm{dt}} = \gamma I$$

Assumptions of simple SIR model: Discretized

- Isolated, well-mixed models work well for small communities
- β: number of infectious contacts by an infectious host per day
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- $\frac{I}{N}$: fraction of hosts who are infectious
  - Assumes all susceptible hosts can be contacted

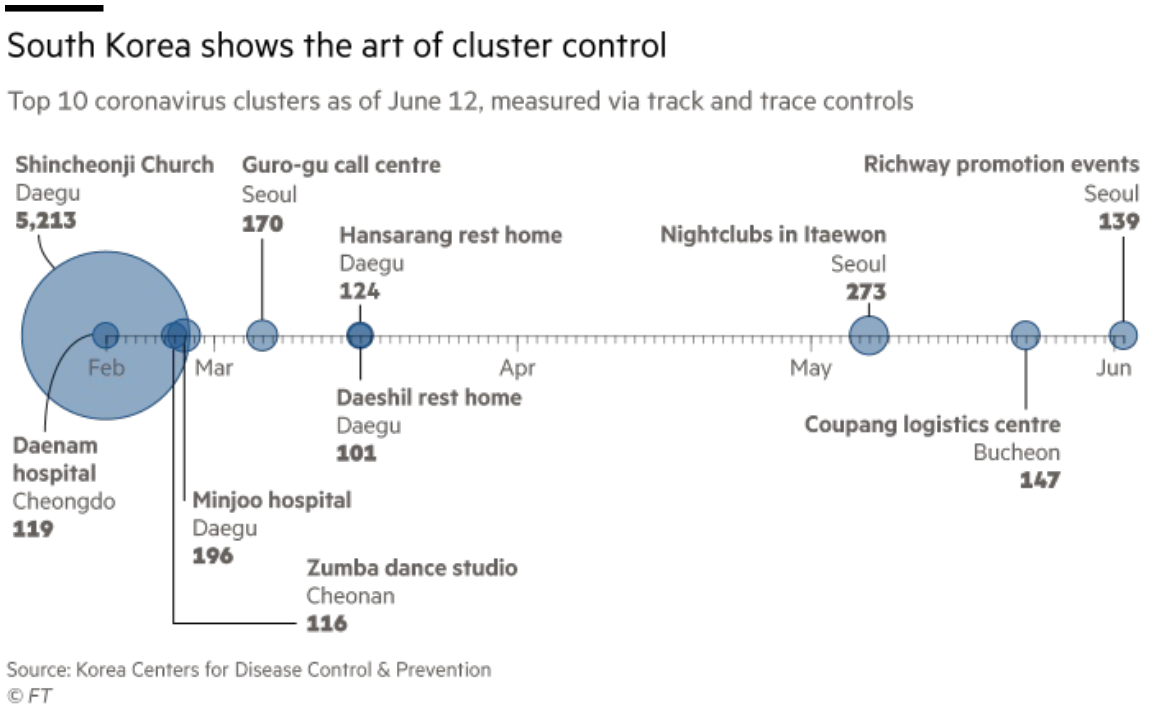
$$S(t+1) = S(t) - \beta S(t)\frac{I(t)}{N}$$

$$I(t+1) = I(t) + \beta S(t)\frac{I(t)}{N} - \gamma I(t)$$

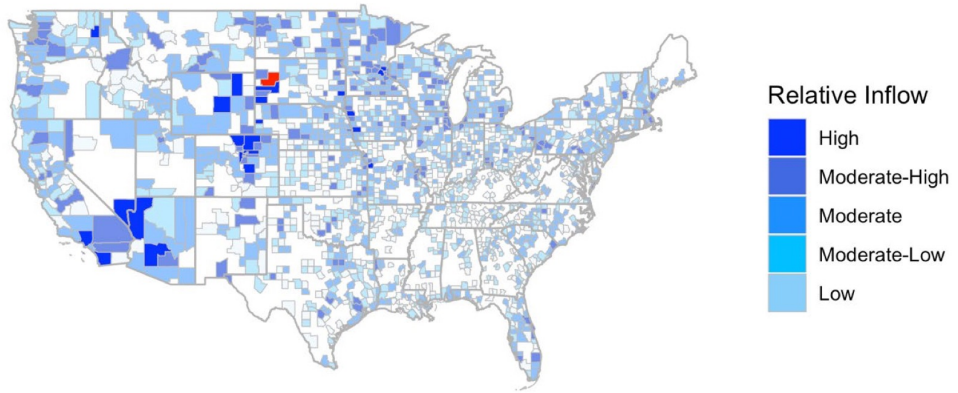
$$R(t+1) = R(t) + \gamma I(t)$$

In reality…

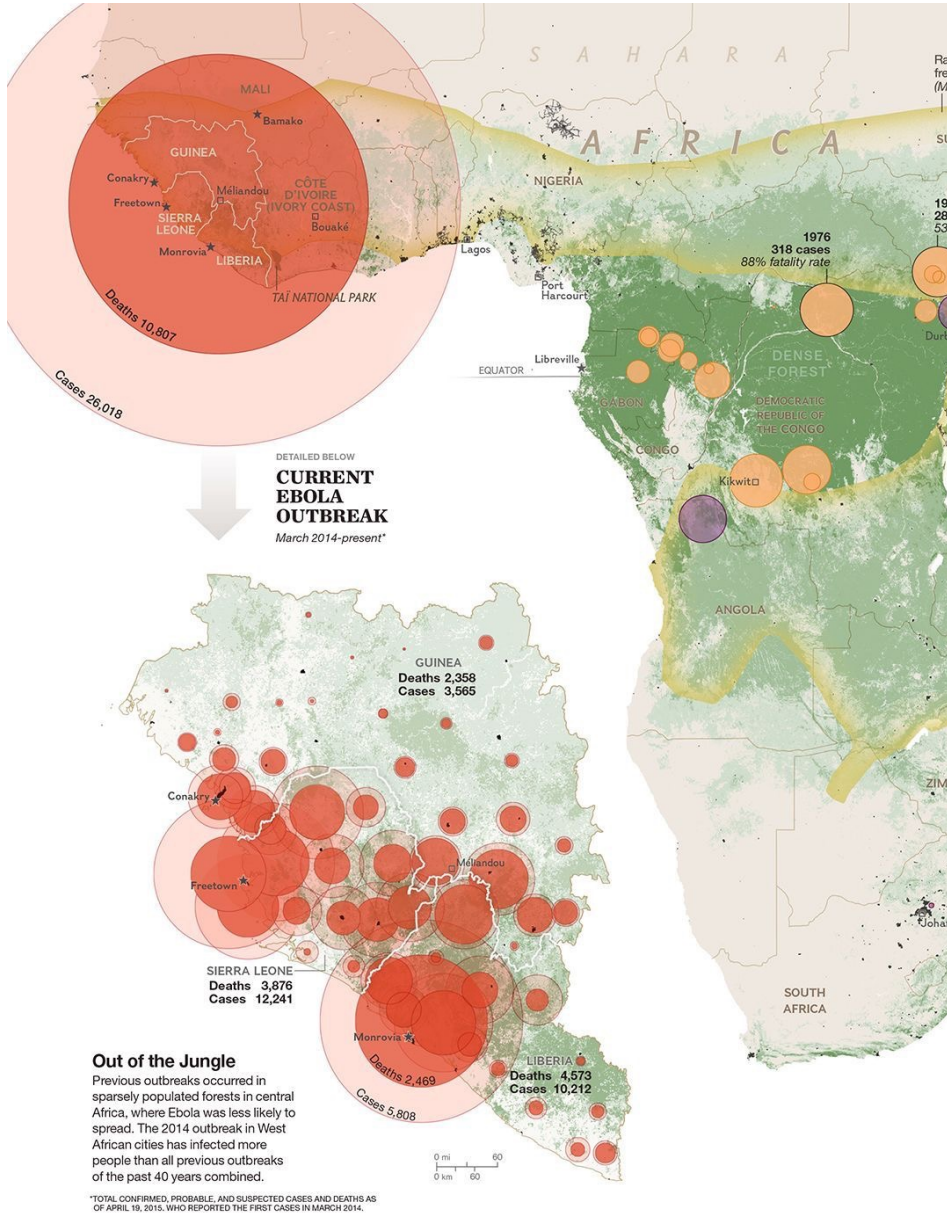
- Risk is highly localized
  - Some people/animals are much more likely to contact than others
- Depends on travel



Panel (b): Relative Intensity of Sturgis Attendee Inflow, by Resident County

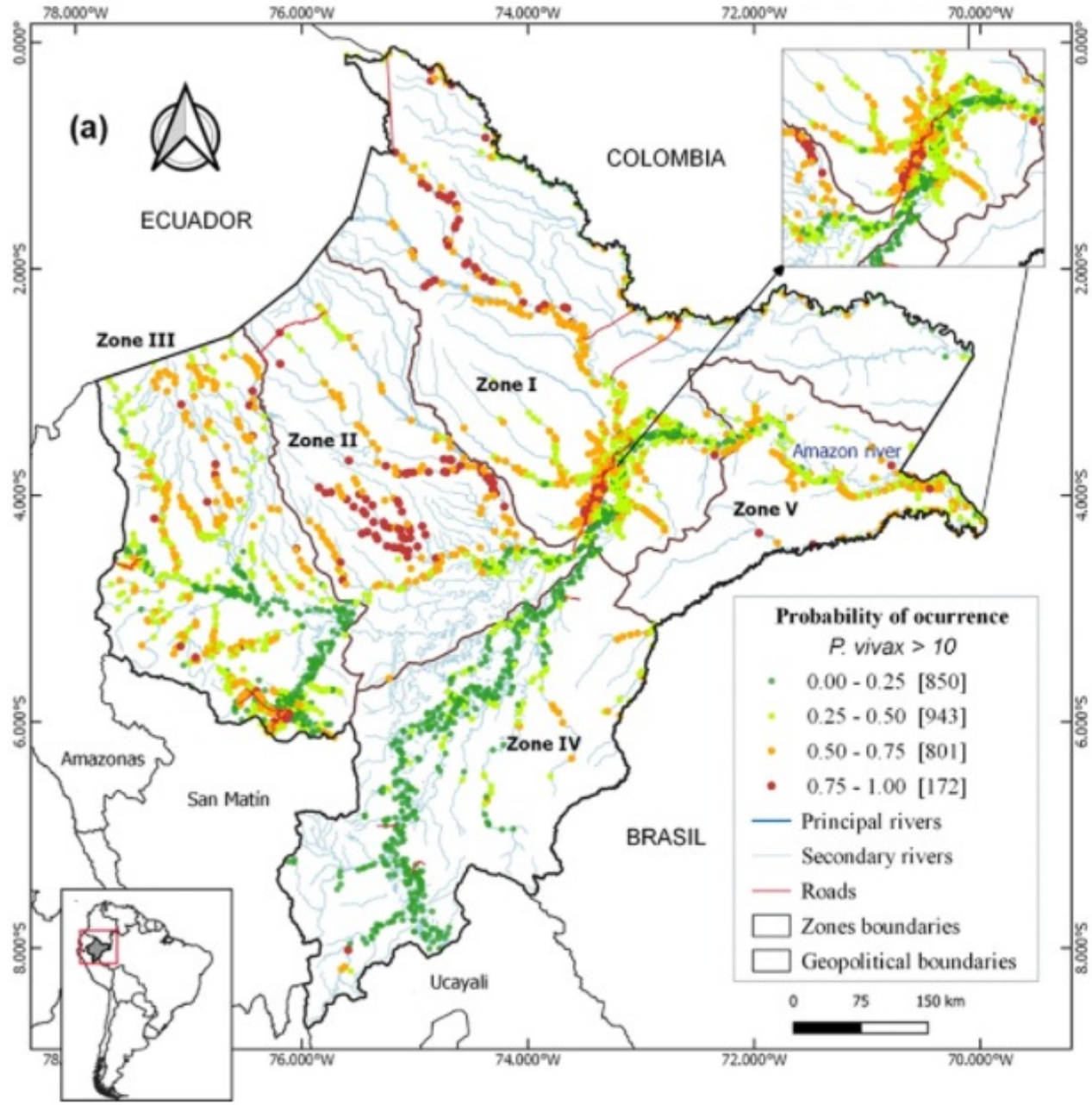


2014 Ebola outbreak



https://www.nationalgeographic.org/activity/mapping-spread-disease-community/

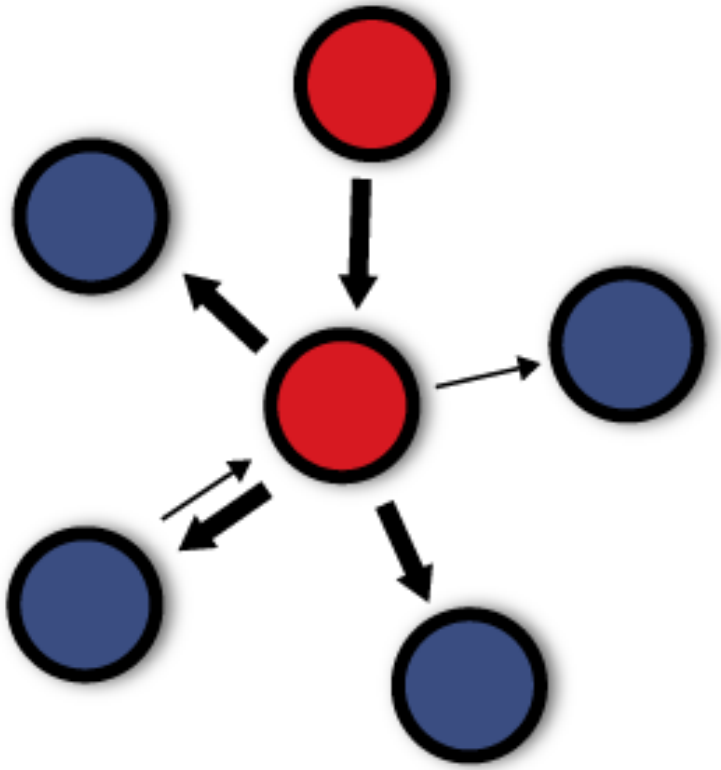
Malaria in Ecuador



https://www.nature.com/articles/s41598-019-51564-4/figures/4

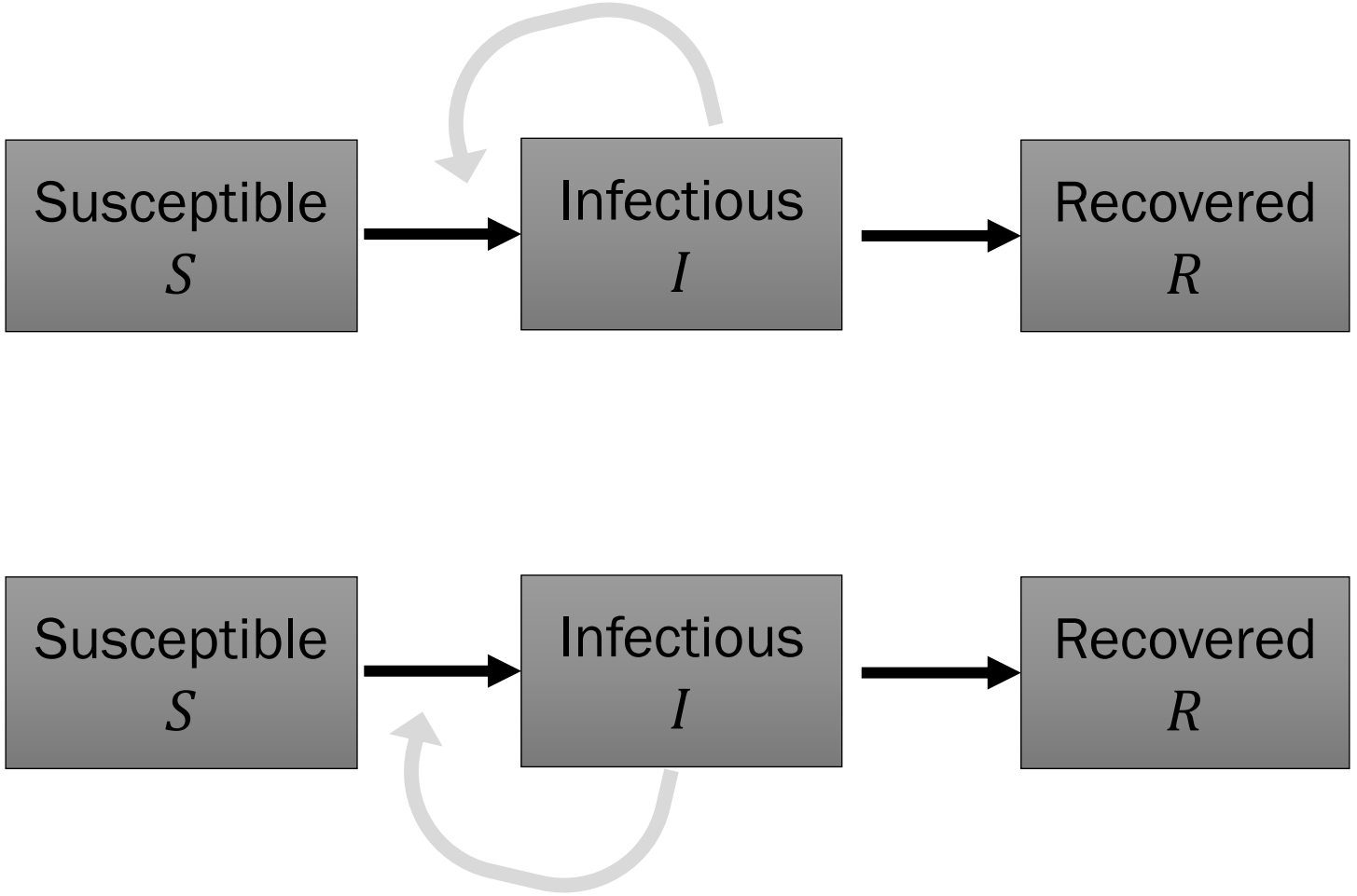
This workshop

- How do we incorporate space into our models?
- When should we?
- What can we learn from spatial models?
- What kinds of analyses do people do?



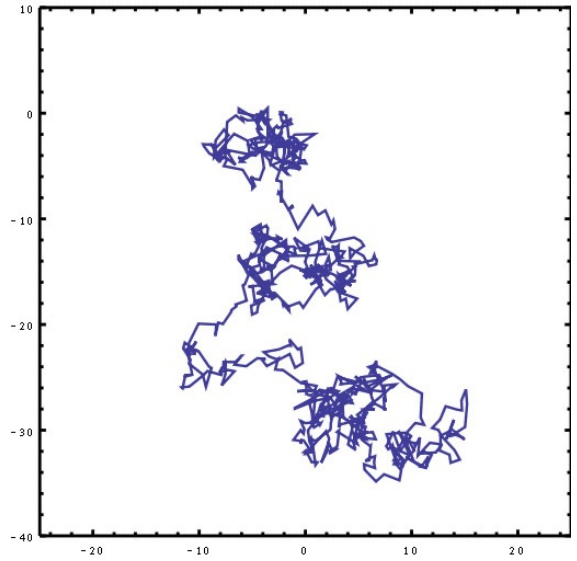
How do we model space?

- S, I, R classes for each community
- Movement between locations
  - Locations can be considered discrete “patches”
- Two ways to account for exposure/mobility of pathogens
  - Physical movement of individuals: Eulerian
  - Exposure during travel: Lagrangian
  - The distinction involves travel



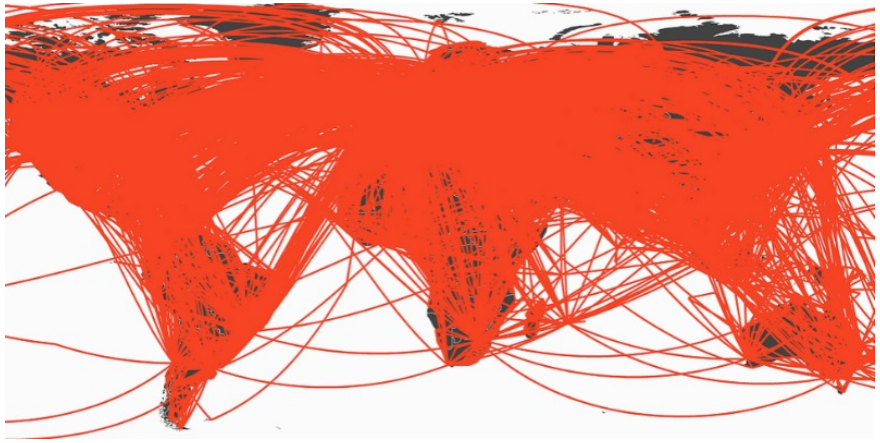
Spatial model methods are often divided by travel patterns

- People usually move short distances rare longer distance movement
- Long distance more common nowadays, though!



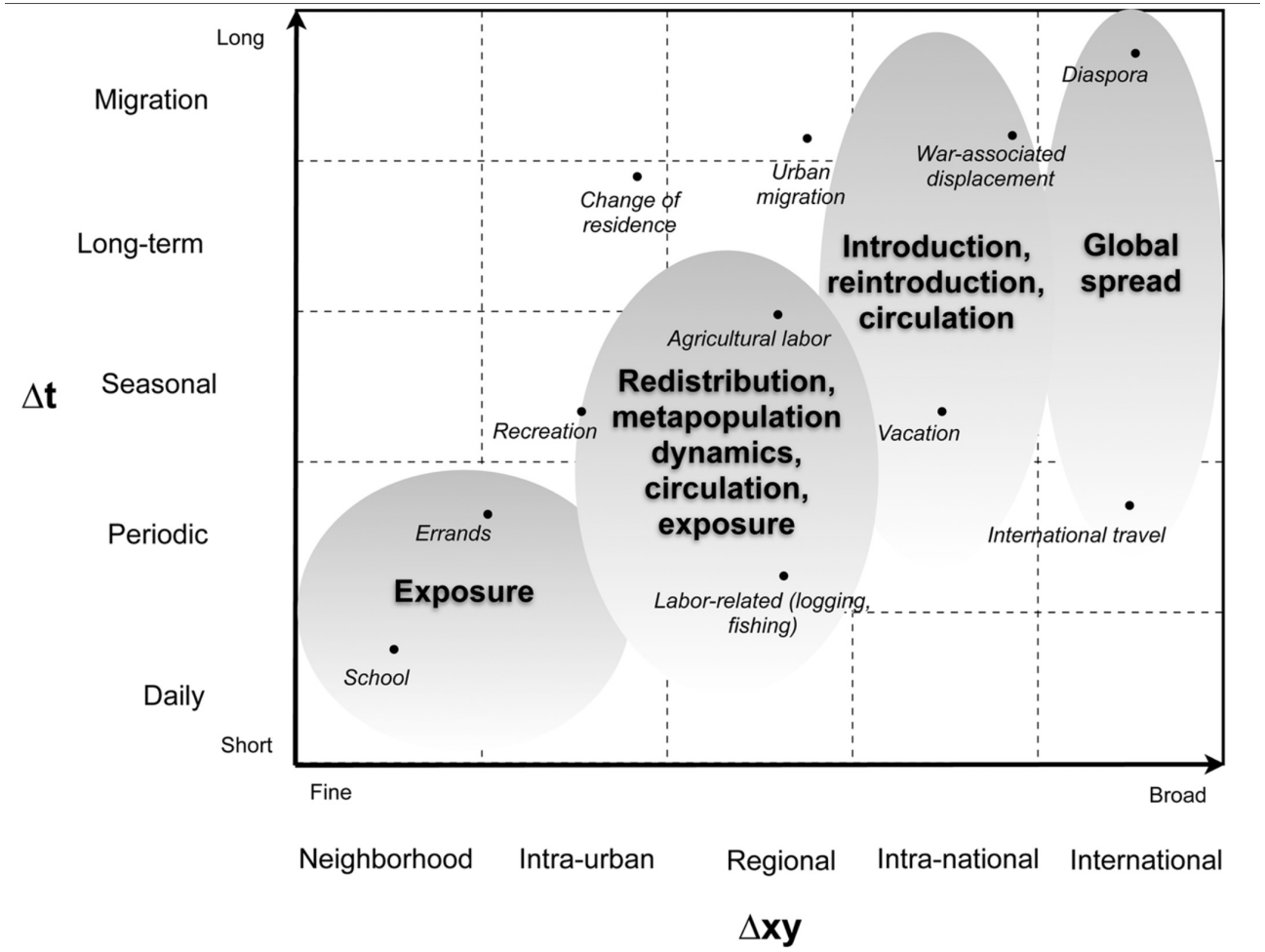
1933

2010



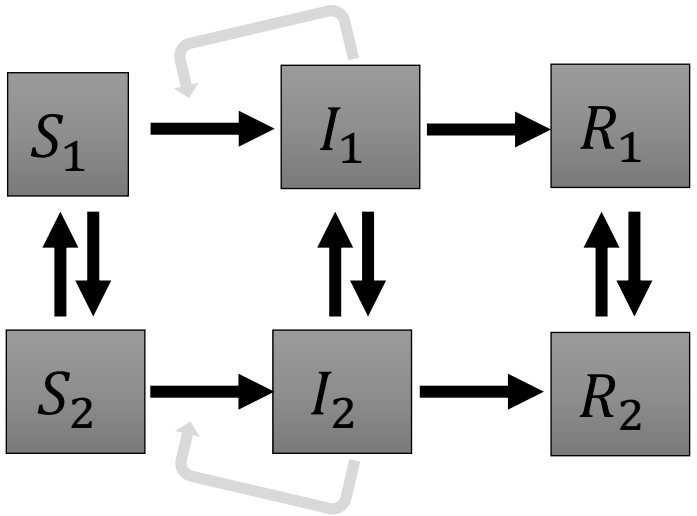
How do we classify it for modeling?

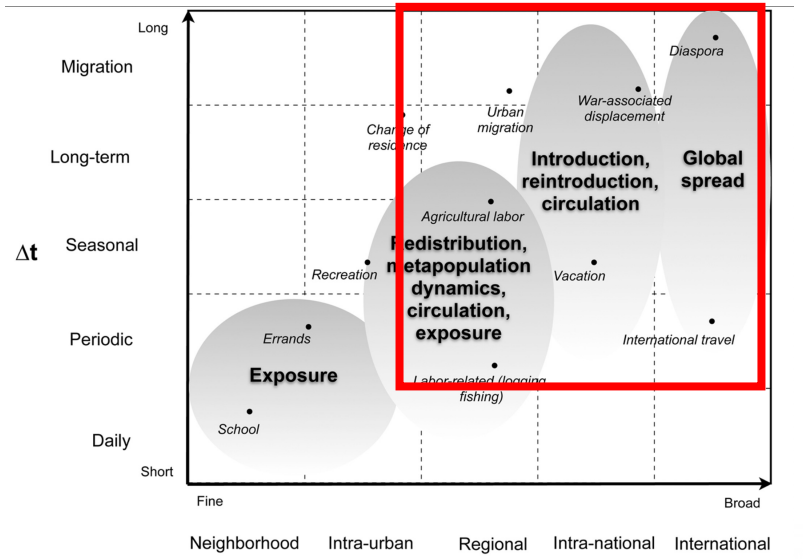
- Two common ways to consider movement:
  - Moving to a new place: “Migration”
  - Being exposed to a new place, but going back home at the end of the day: “Exposure”

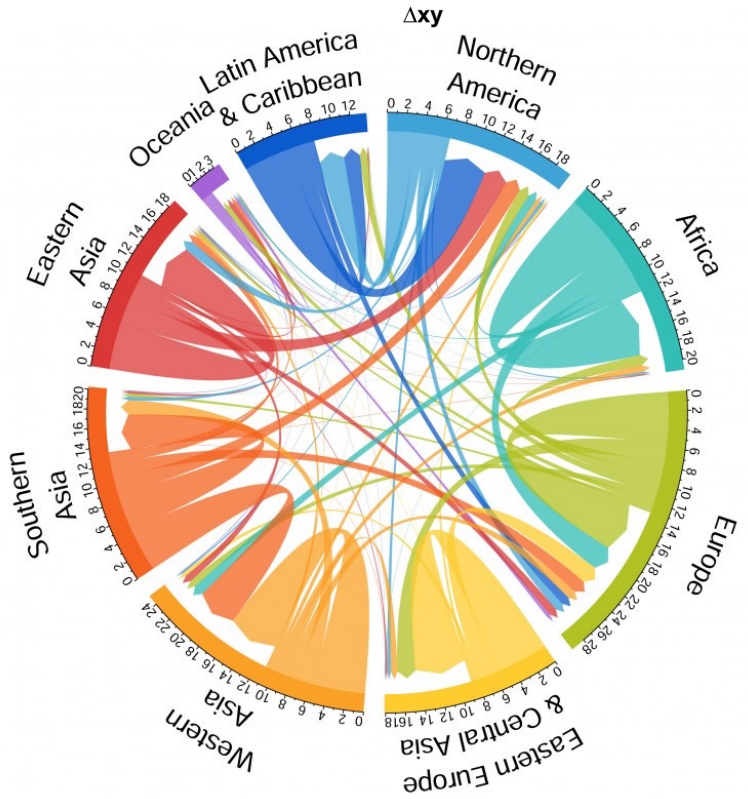


Moving to a new place

- Migration is related to Eulerian movement
  - Move to a new location, and then spend some time there
  - Physically moving individuals from one place to another
- Longer-distance mobility, rarer

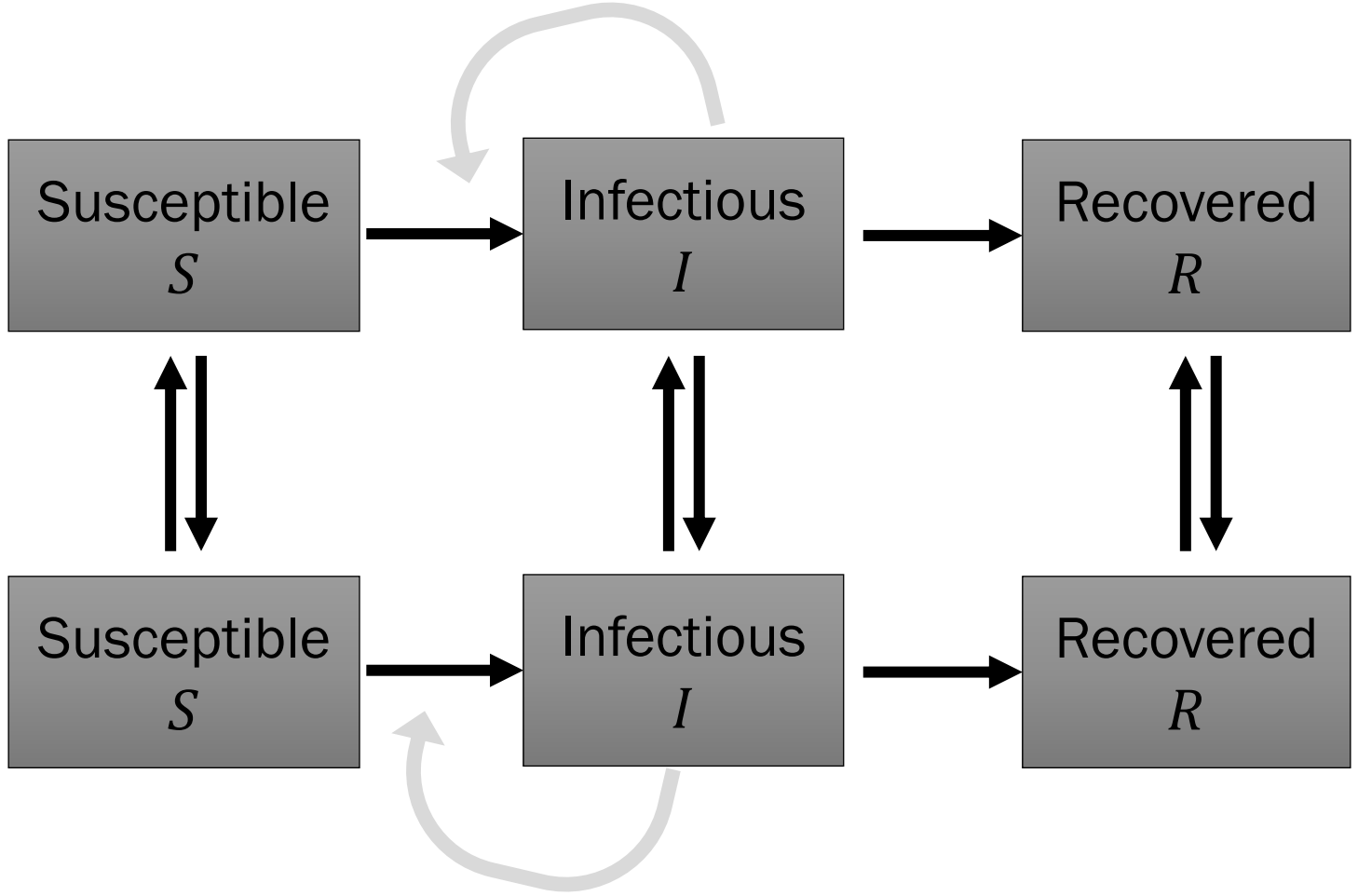






Eulerian movement

- Transmission only happens within the community
- People move physically between them

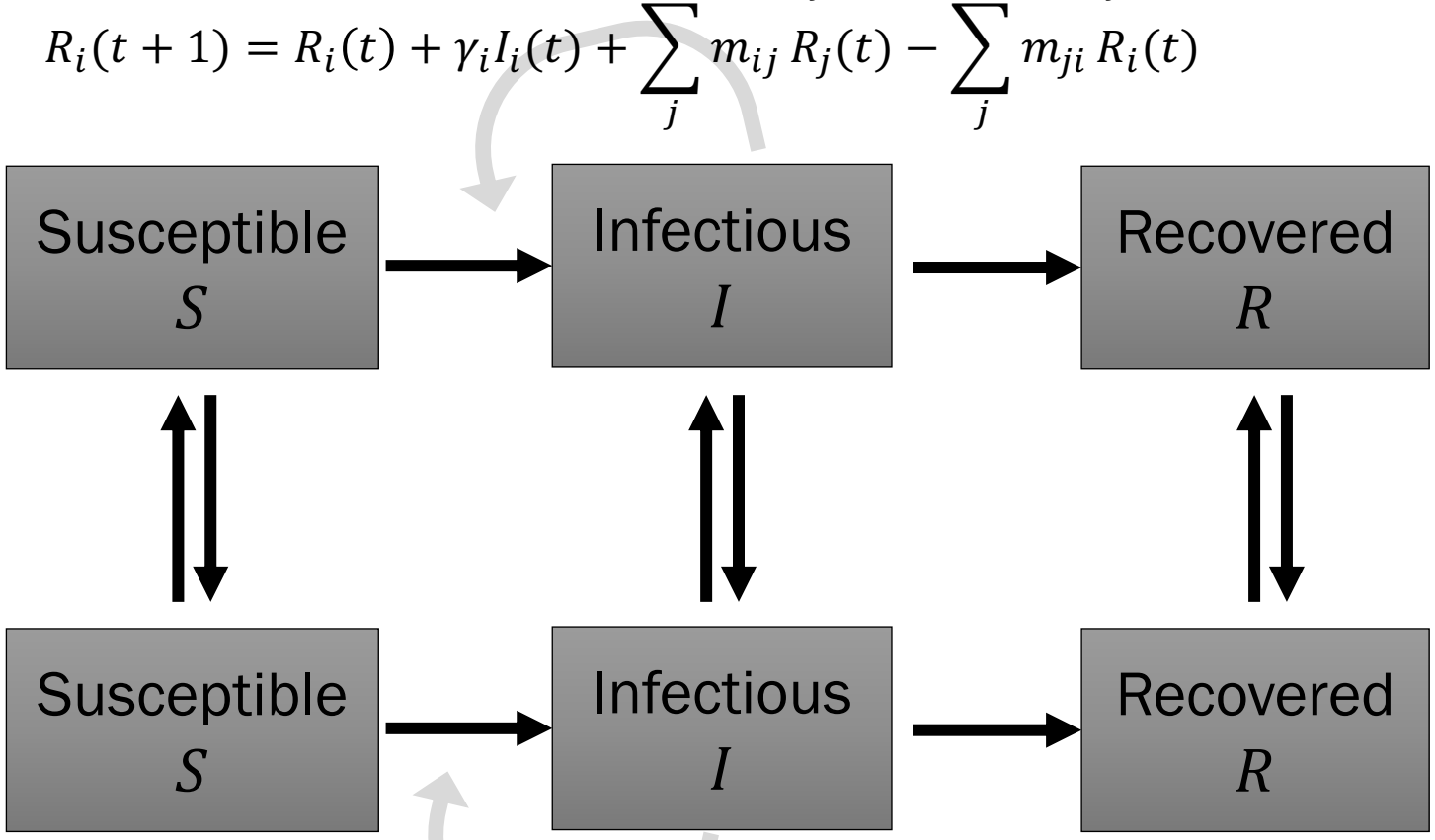


Eulerian movement

$$S_i(t+1) = S_i(t) - \beta_i \frac{S_i(t) I_i(t)}{N_i(t)} + \sum_j m_{ij} S_j(t) - \sum_j m_{ji} S_i(t)$$

$$I_i(t+1) = I_i(t) + \beta_i \frac{S_i(t) I_i(t)}{N_i(t)} - \gamma_i I_i(t) + \sum_j m_{ij} I_j(t) - \sum_j m_{ji} I_i(t)$$

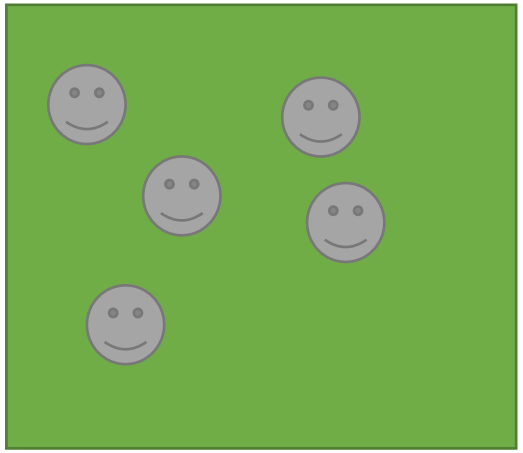
$$R_i(t+1) = R_i(t) + \gamma_i I_i(t) + \sum_j m_{ij} R_j(t) - \sum_j m_{ji} R_i(t)$$

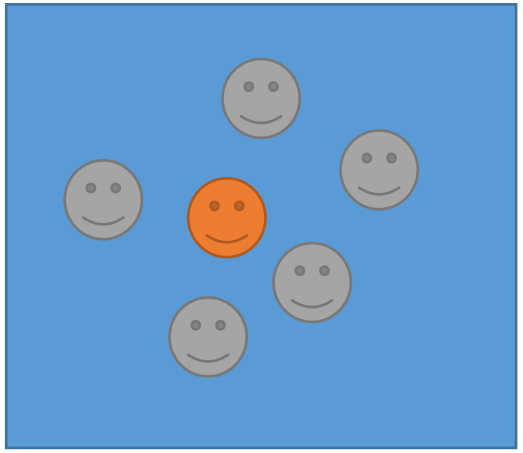


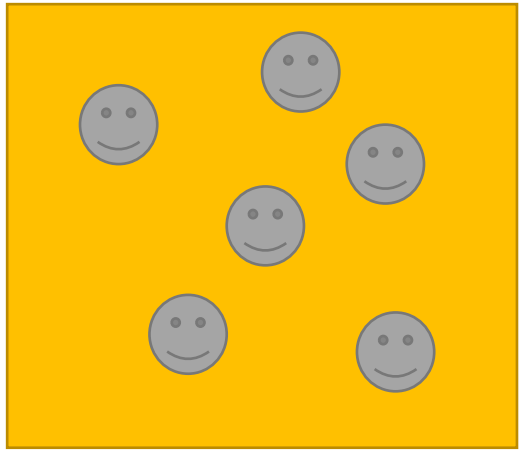
Physically moving

Susceptible

Infected



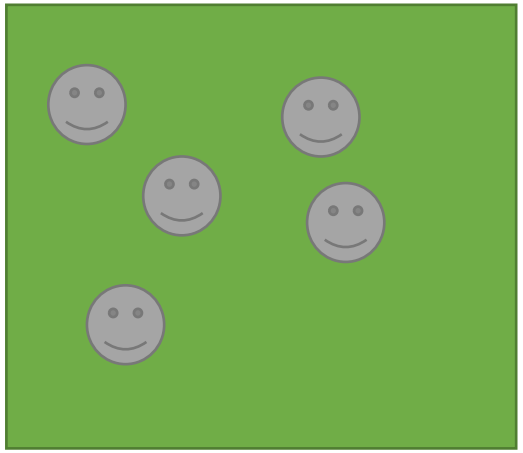


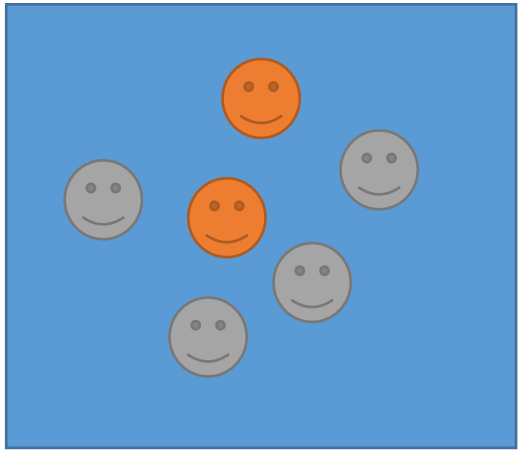


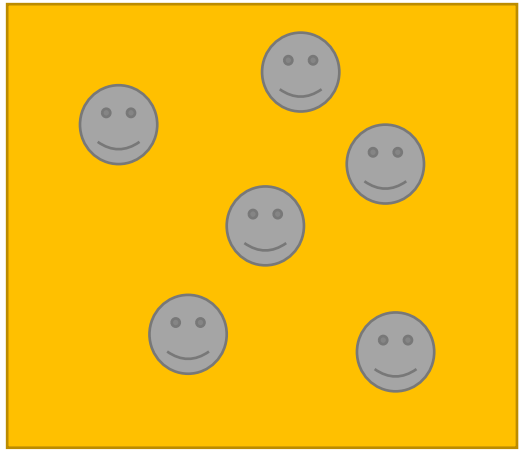
Moving to a new place

Susceptible

Infected







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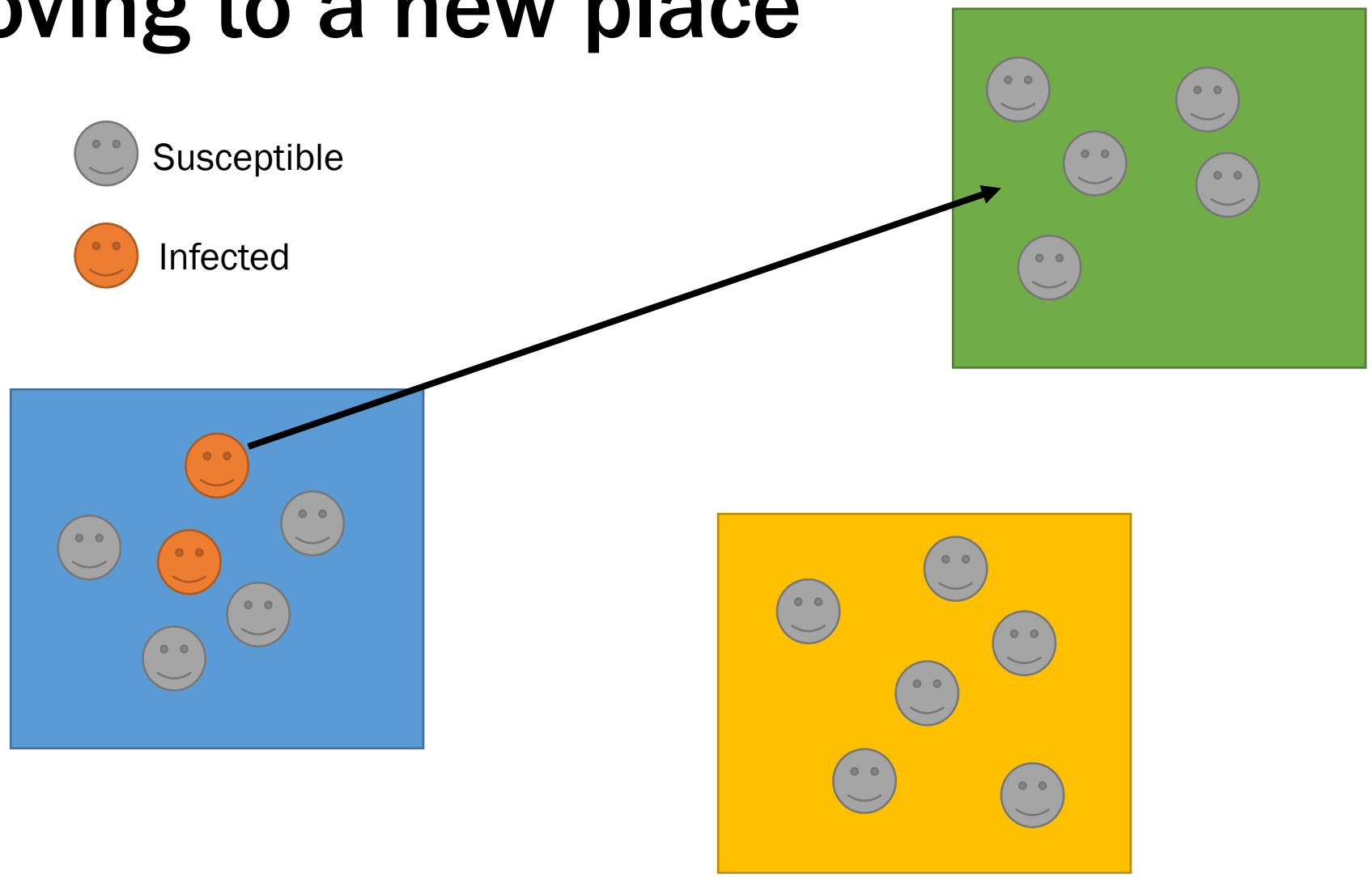

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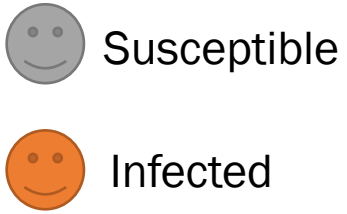

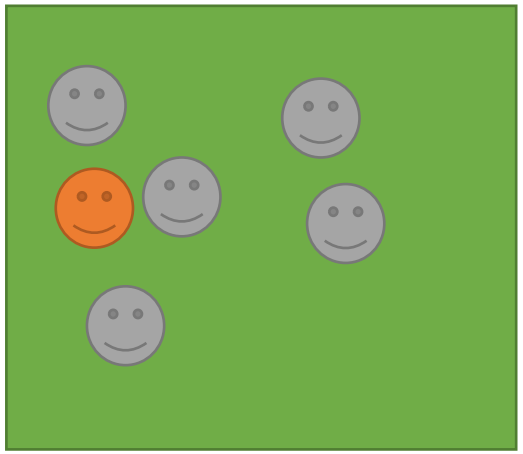

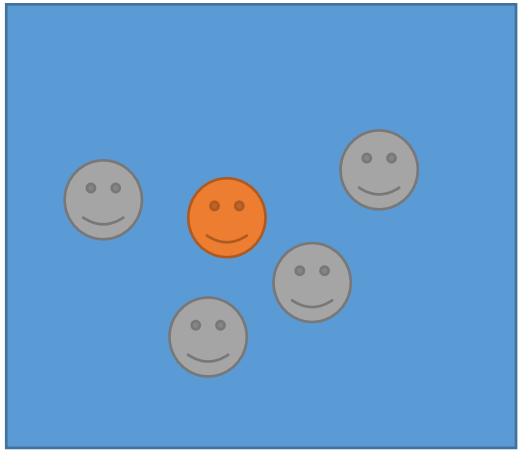

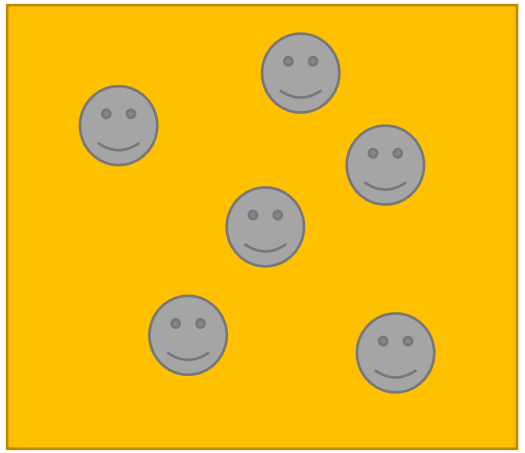

Moving to a new place


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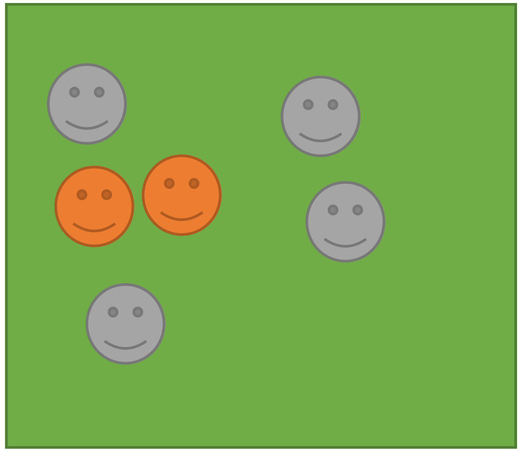

$$S(t+1) = S(t) - \beta S(t)\frac{I(t)}{N}$$
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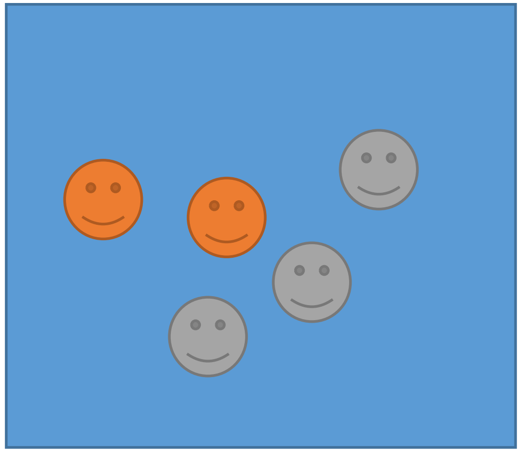

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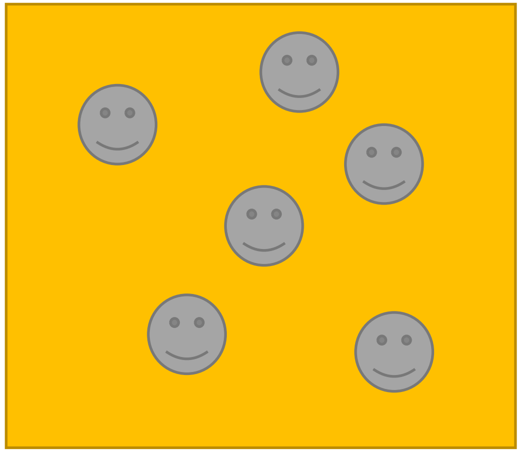

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Modifying our model for migration

- Migration is added as a separate process from transmission, recovery
- Each population (S/I/R) moves to new patches at a rate
  - m_{21}: Rate of people per day moving from patch 1 to patch 2
- Bidirectional
  - m_{12}: Rate of people per day moving from patch 2 to patch 1

$$S(t+1) = S(t) - \beta S(t)\frac{I(t)}{N}$$

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Moving to a new place

- Transmission as usual, but people move in and out (staying in the same class)
- $S_1(t+1) = S_1(t) - \beta S_1(t)\frac{I_1(t)}{N_1(t)} + m_{12}S_2(t) - m_{21}S_1(t)$

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Moving in

Moving out

m_{12}=0.1 means for every person in S_2 0.1 person moves from patch 2 to patch 1 per day

Also means people stay in patch 2 for 1/0.1, or 10 days, on average

Eulerian movement

$$S_i(t+1) = S_i(t) - \beta_i \frac{S_i(t) I_i(t)}{N_i(t)} + \sum_j m_{ij} S_j(t) - \sum_j m_{ji} S_i(t)$$

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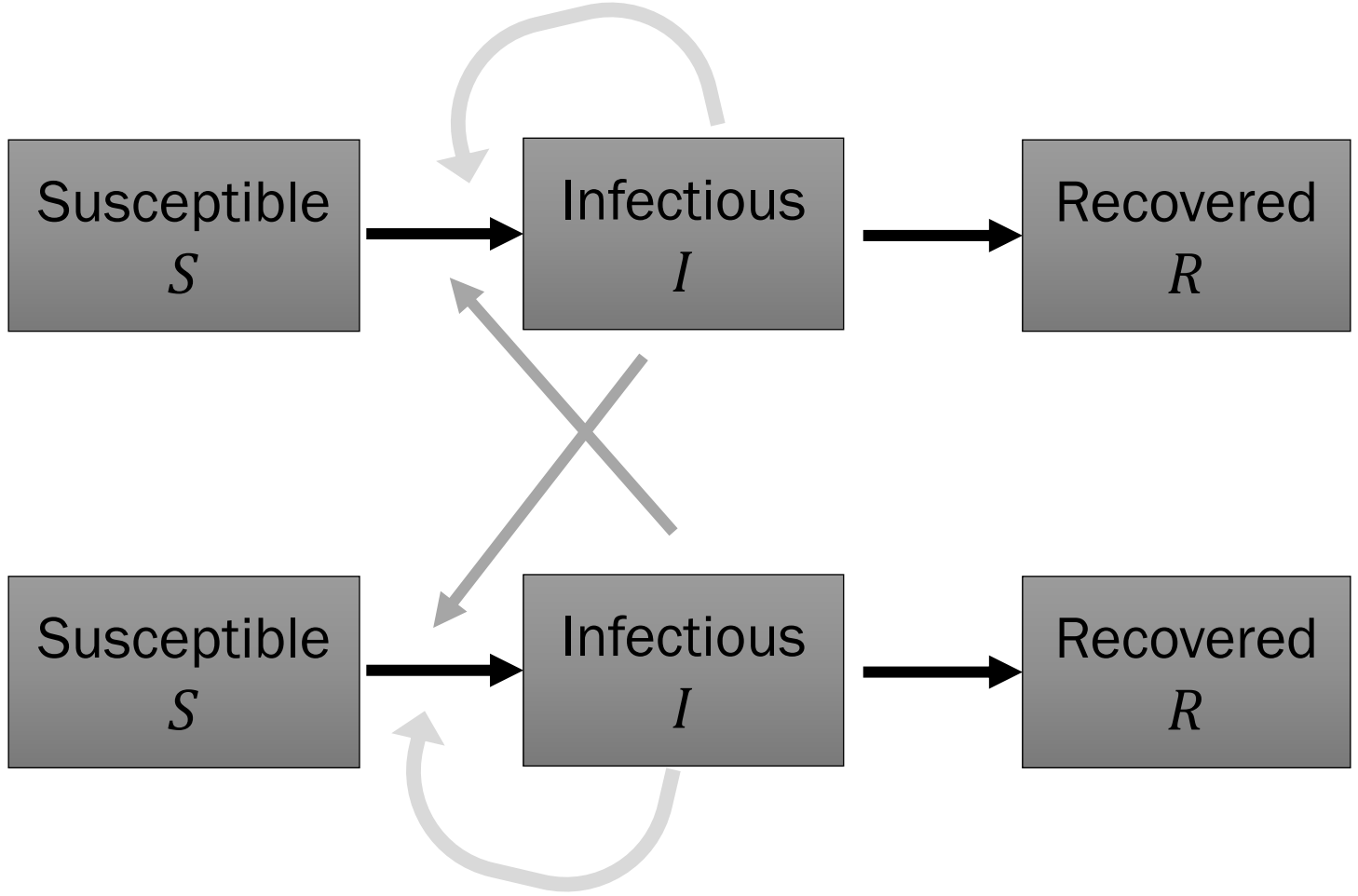
$$R_i(t+1) = R_i(t) + \gamma_i I_i(t) + \sum_j m_{ij} R_j(t) - \sum_j m_{ji} R_i(t)$$

- We move people physically from one area to another
  - Some S_1 individuals move to patch 2 and become S_2, etc
- Transmission only occurs within patch
  - I_2 individuals can only infect S_2
- What else?

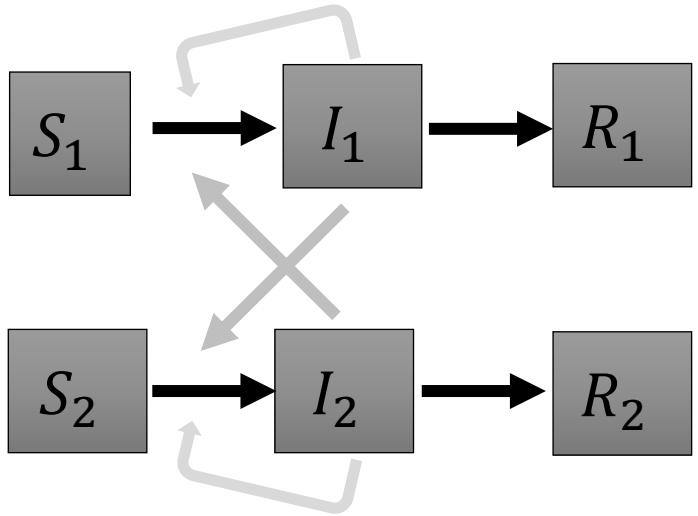
Lagrangian movement

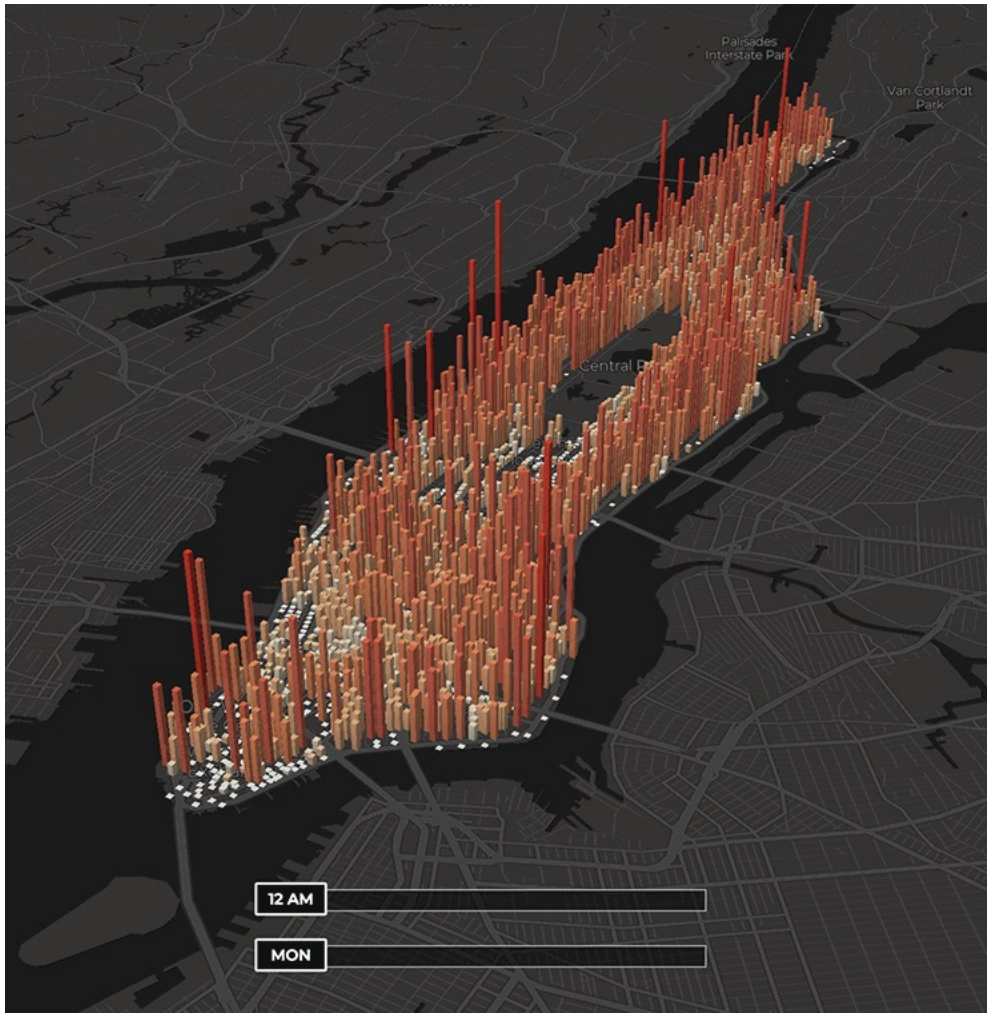
- Infectious people in one patch contact susceptible people in another
  - Cross-community transmission will be much lower
  - Exposure changes with time spent in different places



Changing exposure

- Associate people with a resident patch, but model exposure in other patches
  - Always associated with home patch
- Shorter-distance spread and mobility: Neighborhoods, nearby cities

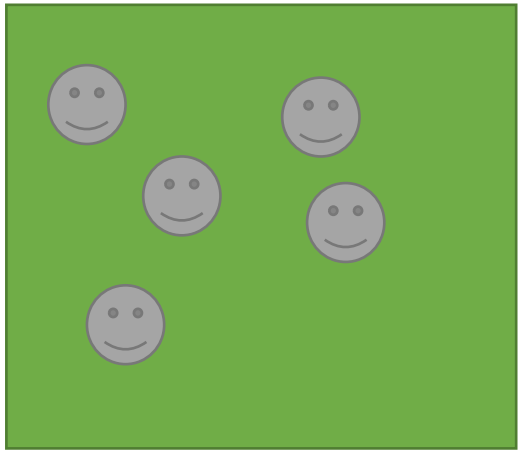


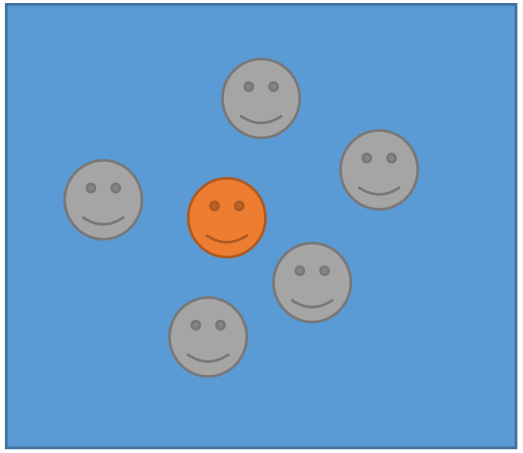


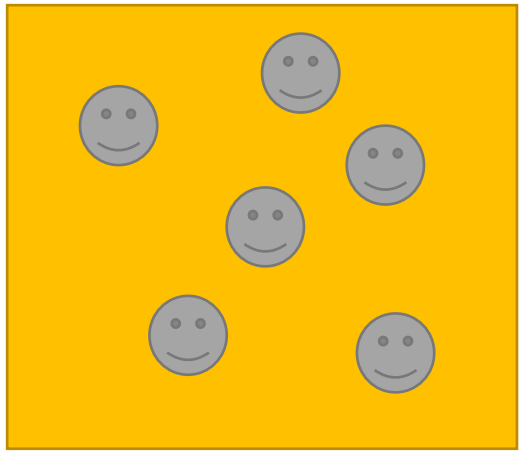
Changing exposure

Susceptible

Infected







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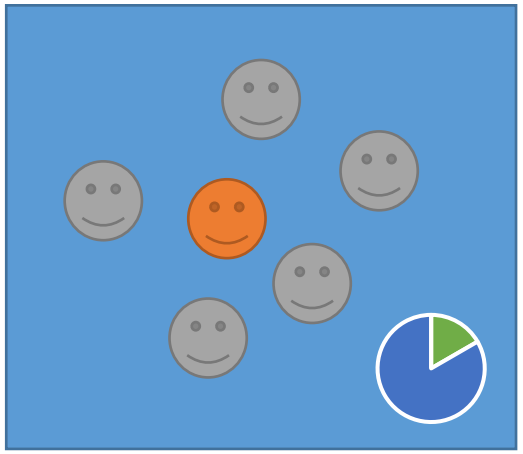
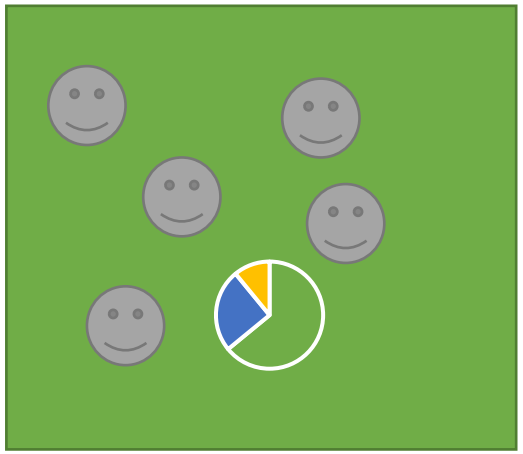
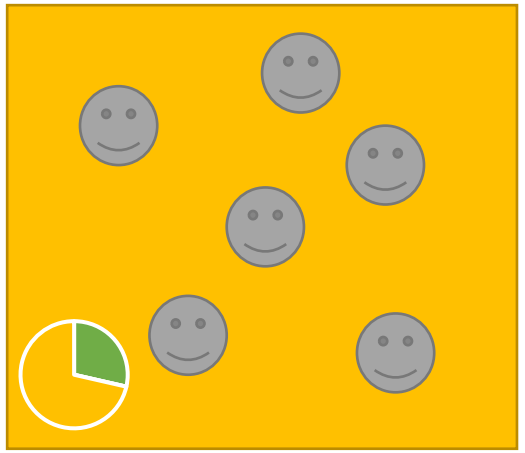
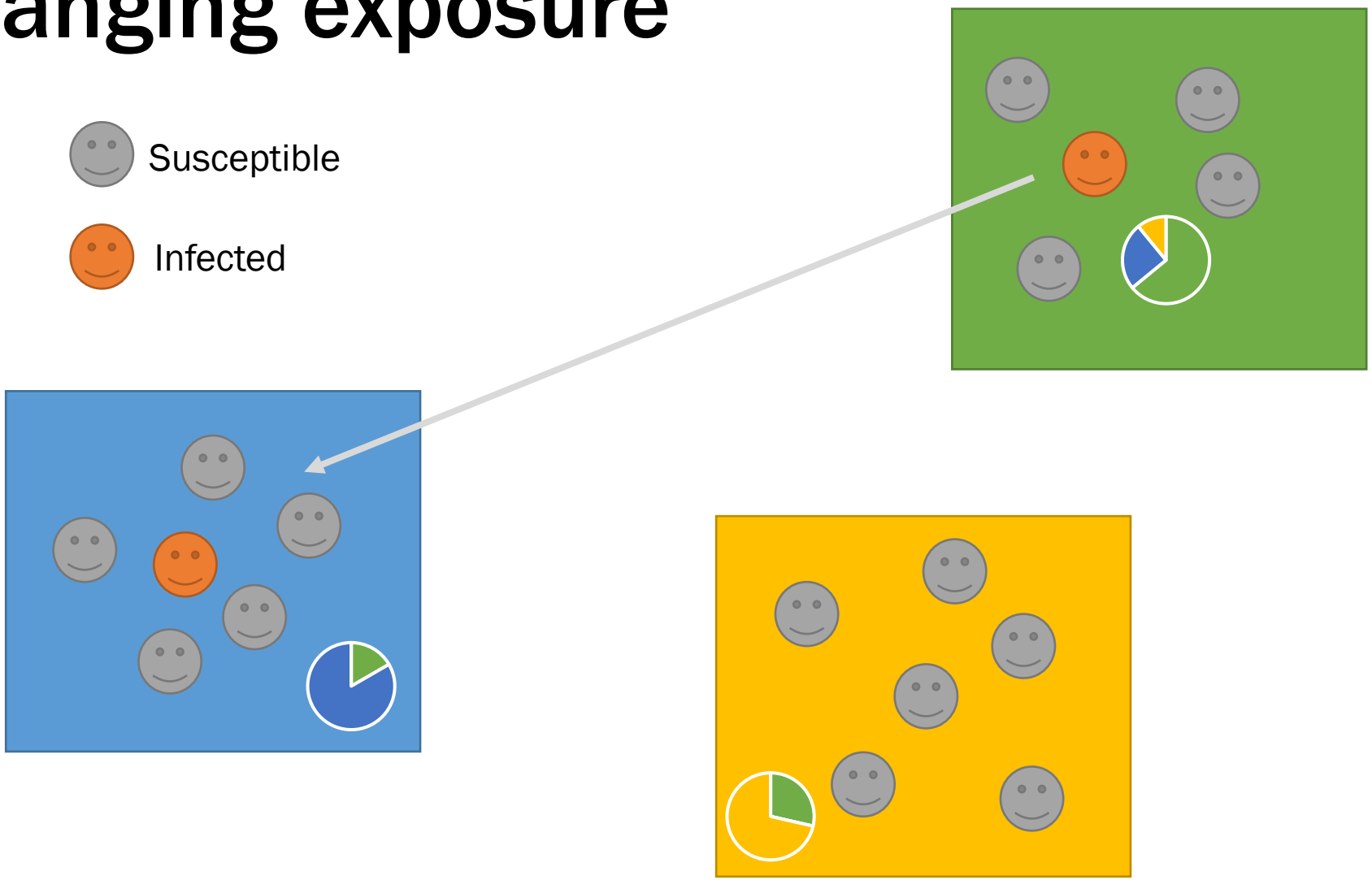






Changing exposure



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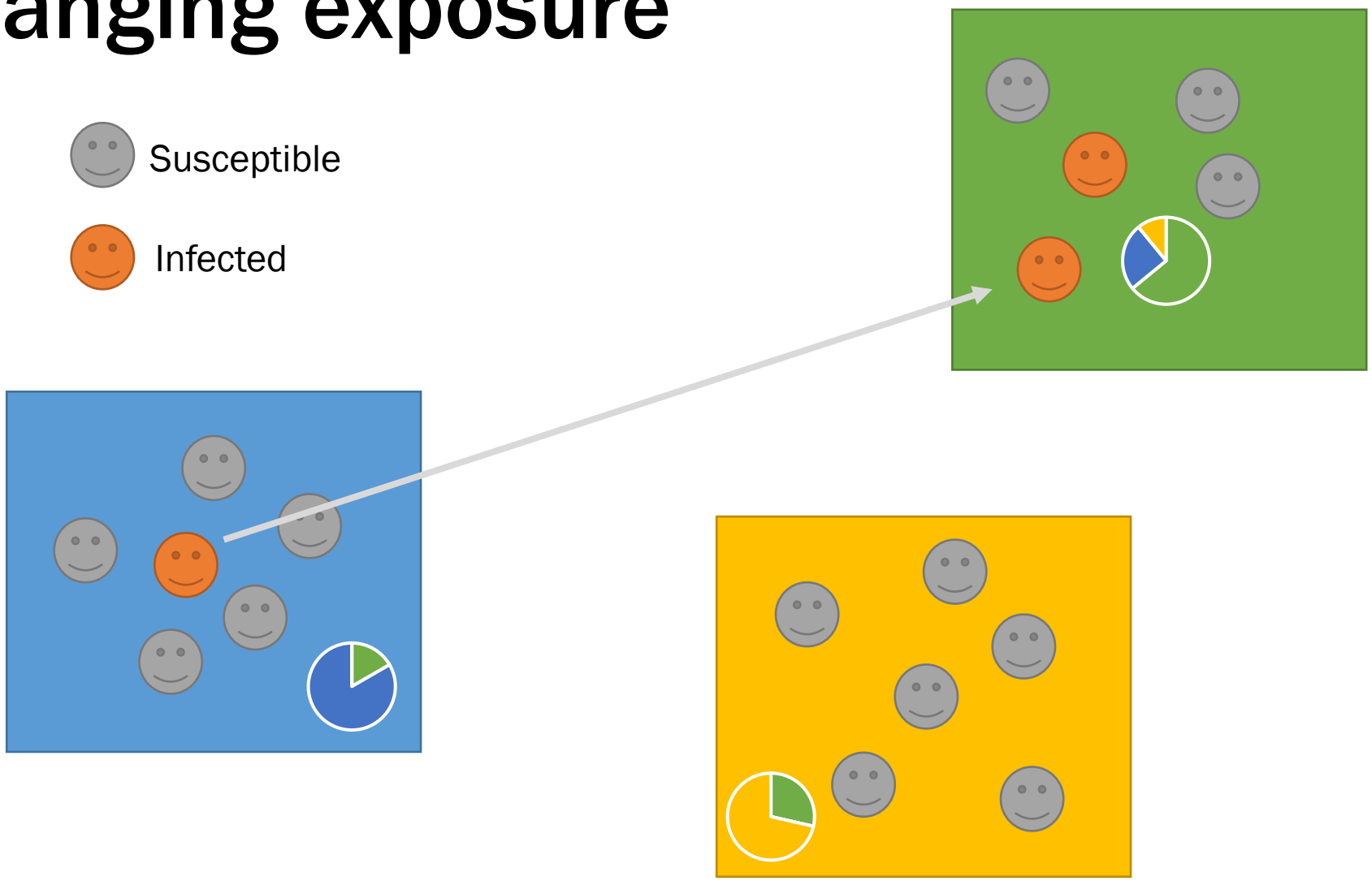

Changing exposure

$$S_1(t+1) = S_1(t) - \beta S_1(t)\frac{I_1(t)}{N_1}$$

$$I_1(t+1) = I_1(t) + \beta S_1(t)\frac{I_1(t)}{N_1} - \gamma I_1(t)$$

$$R_1(t+1) = R_1(t) + R_1(t) + \gamma I_1(t)$$

- How do we include the effect of exposure to infectious populations in other places?
  - Does NOT involve explicit movement

Changing exposure

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Here, transmission is coming entirely from infectious people in patch 1

- How do we include the effect of exposure to infectious populations in other places?
  - Does NOT involve explicit movement

Changing exposure

$$\beta S_1 \frac{I_1}{N_1}$$

- Change I_1 to be a weighted average of where people from patch 1 spend time
- If 50% patch 1, 25% patch 2, 25% patch 3…

$$\beta \left(0.5 * \frac{I_1}{\widehat{N}_1} + 0.25 * \frac{I_2}{\widehat{N}_2} + 0.25 * \frac{I_3}{\widehat{N}_3} \right) S_1$$

- Where $\widehat{N}_1 = p_{11}N_1 + p_{12}N_2 + p_{13}N_3$

Changing exposure

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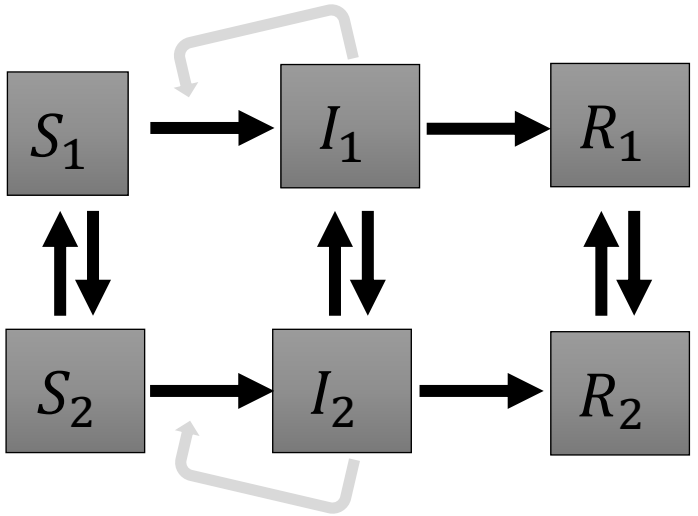
$$\beta \left(\boxed{0.5} * \frac{I_1}{\widehat{N}_1} + \boxed{0.25} * \frac{I_2}{\widehat{N}_2} + \boxed{0.25} * \frac{I_3}{\widehat{N}_3} \right) S_1$$

50% of exposure is to I_1

25% of exposure is to I_2

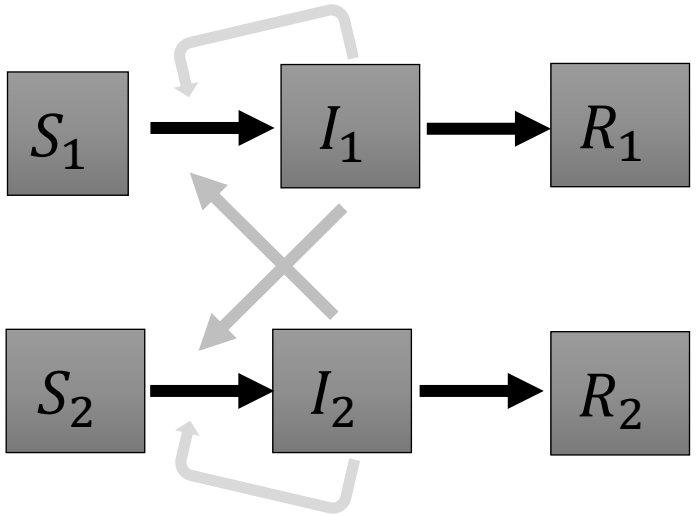
25% of exposure is to I_3

- Where $\widehat{N}_1 = p_{11}N_1 + p_{12}N_2 + p_{13}N_3$



- We move people physically from one area to another
  - Some S_1 individuals move to patch 2 and become S_2, etc
- Transmission only occurs within patch
  - I_2 individuals can only infect S_2

"Eulerian" movement

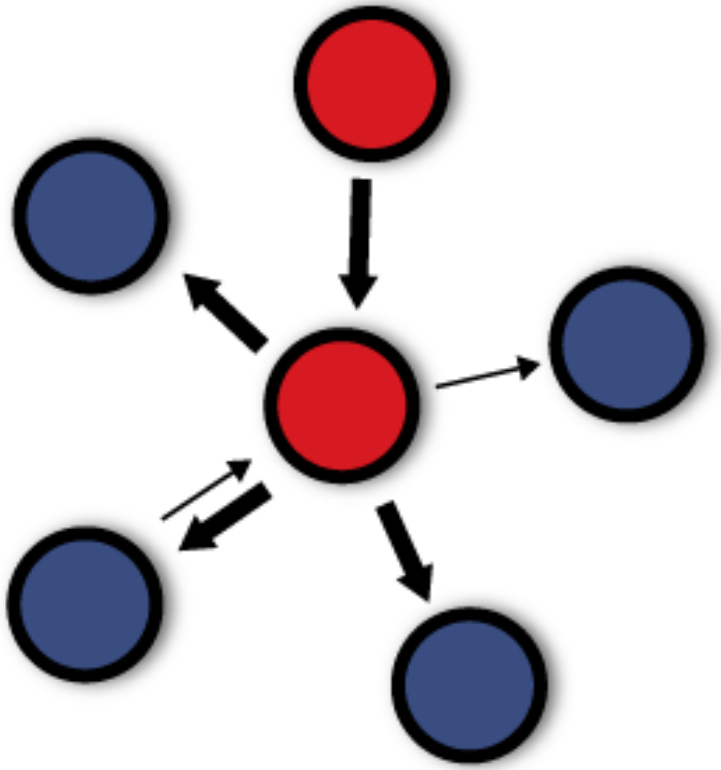


- People always stay in the same place
- Cross-community transmission occurs
  - I_2 can infect S_1, but probably at a much lower contact rate than i_2 to S_2

"Lagrangian" movement

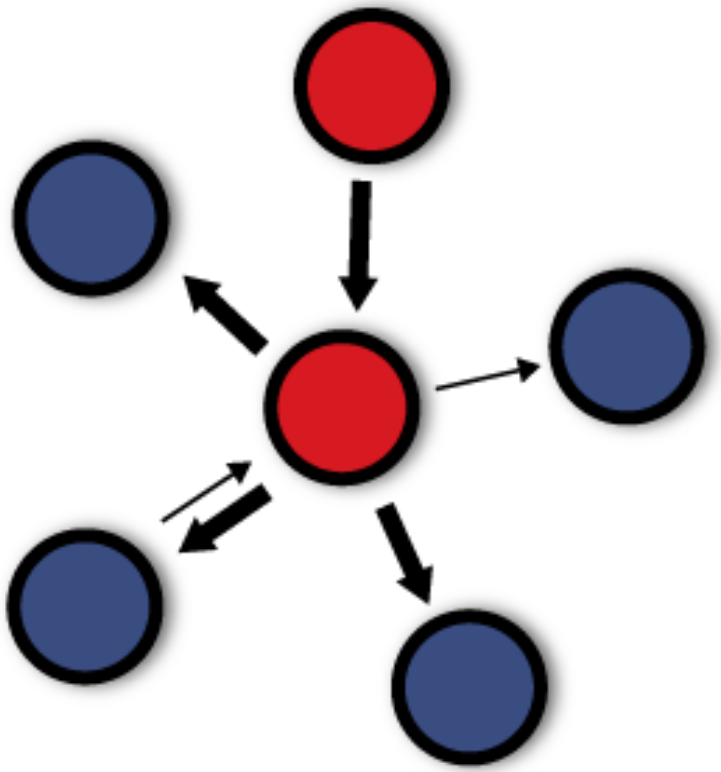
When would we want to model space?

- exposure risk varies across locations
- movement links otherwise separate populations
- interventions or surveillance differ across space
- importation vs. local transmission matters
- persistence depends on source/sink dynamics
- the research question is about *where*, *when*, or *through which routes* transmission occurs.

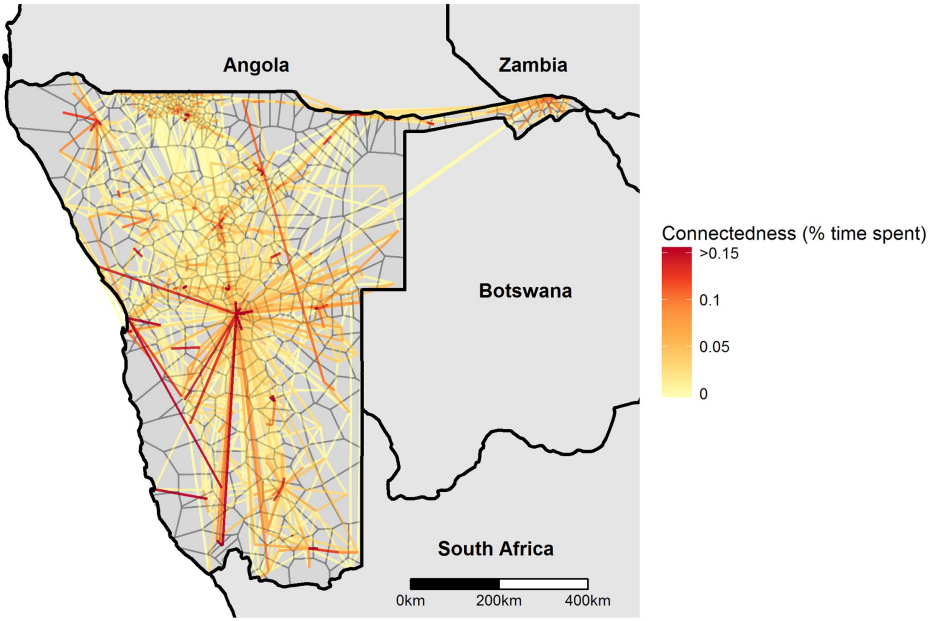


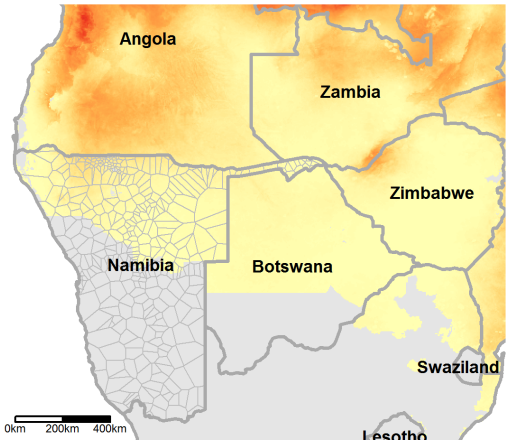
What can we do in these heterogeneous environments?

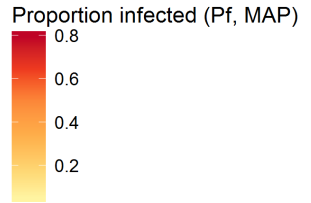
- People move pathogens between communities/areas
  - Creates interesting dynamics that can be explored using mathematical modelling approaches
    - Source/sink dynamics
    - Intervention targeting
    - Optimal control

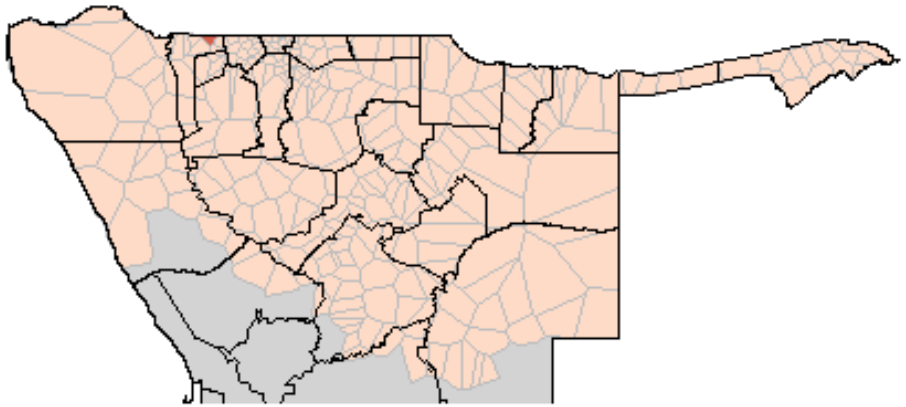


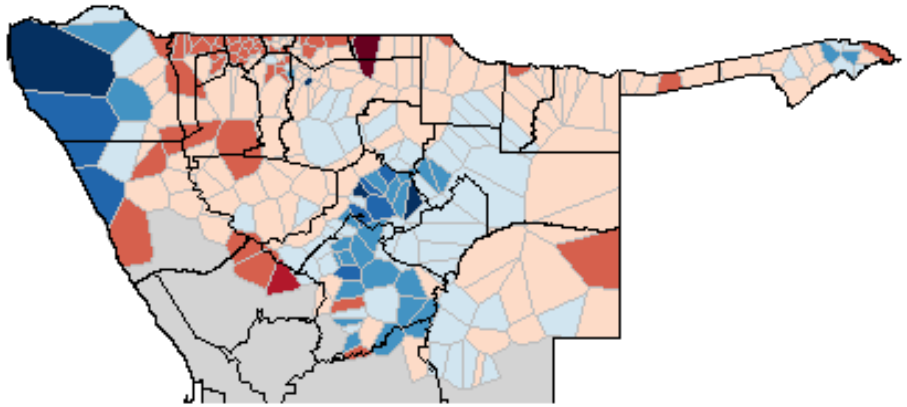
Source/sink dynamics

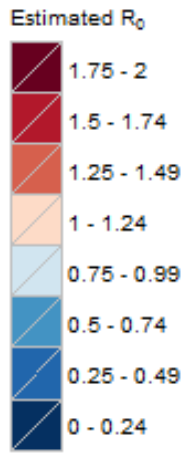




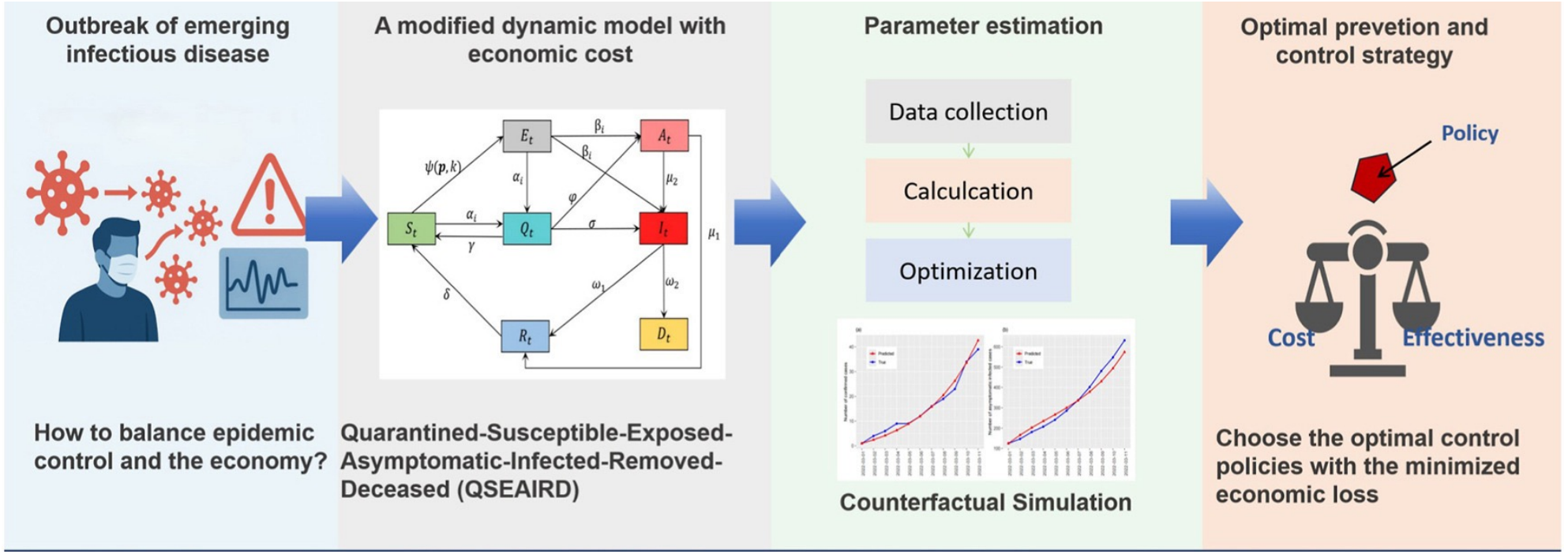








Optimal control

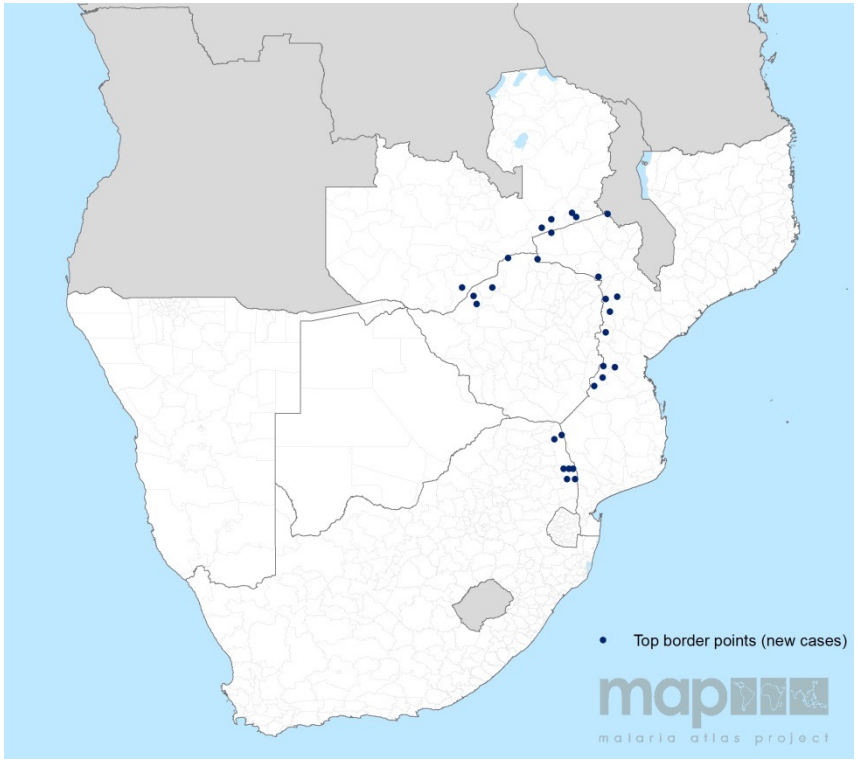


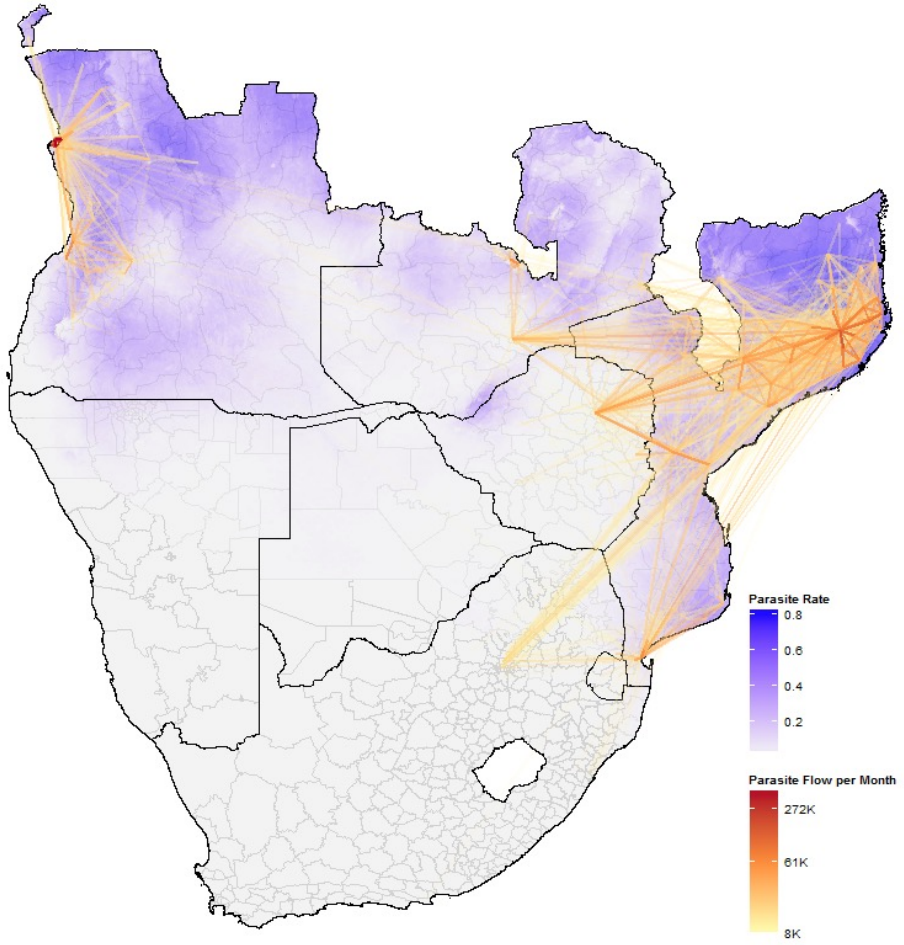
- Define objective function: costs, desired outcome variable
- Minimize objective function based on timing/placement of interventions

https://www.sciencedirect.com/science/article/pii/S2468042725001149

Targeted interventions

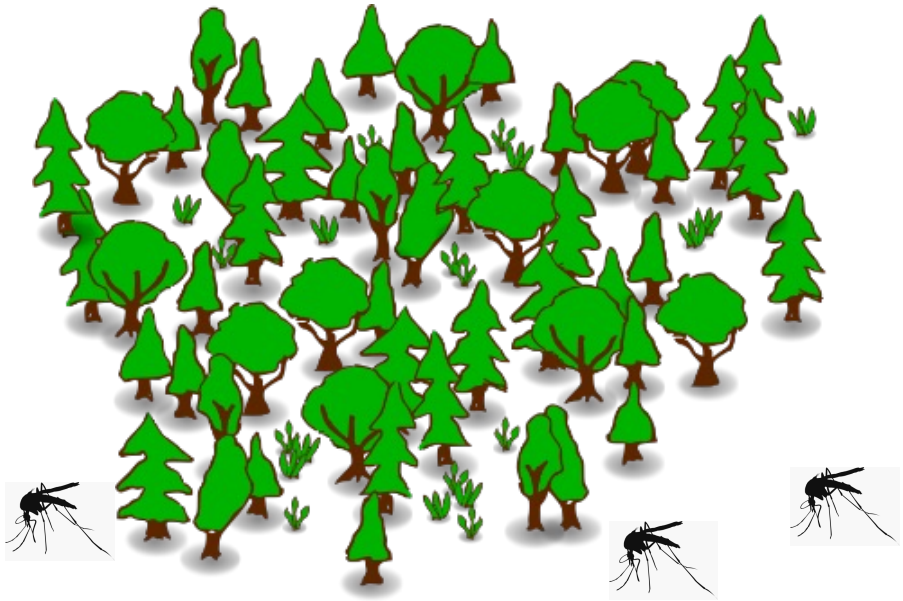
- Top 28 facility locations within 60 km of an E8-E8 border that could prevent the most new cases that are not already served by a facility



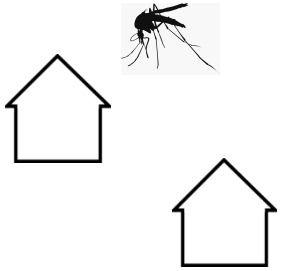


Codifying spatial targeting

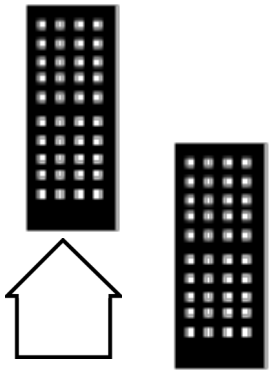
Malaria hotspot



Populated areas near hotspot



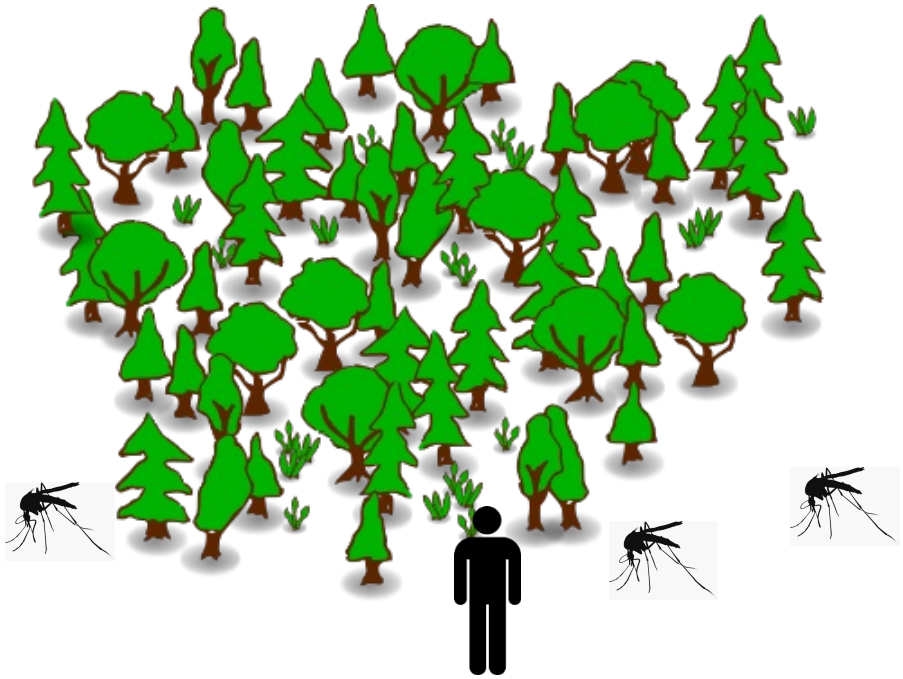
Distant urban areas



Different stages of the infectious disease spread process warrant different interventions

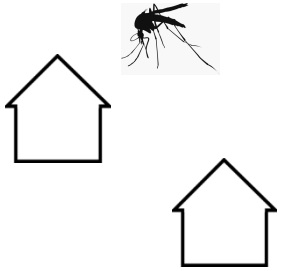
Codifying spatial targeting

Malaria hotspot



Step 1: Initial infection

Populated areas near hotspot



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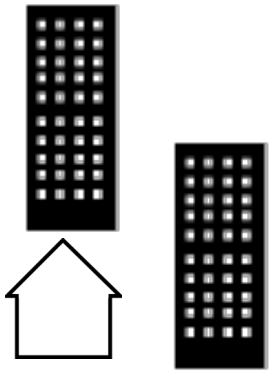

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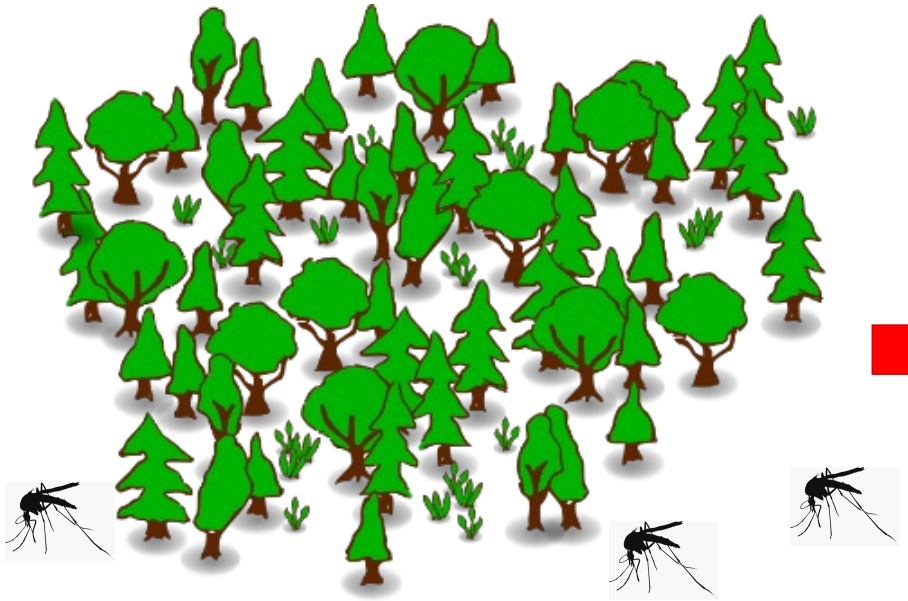
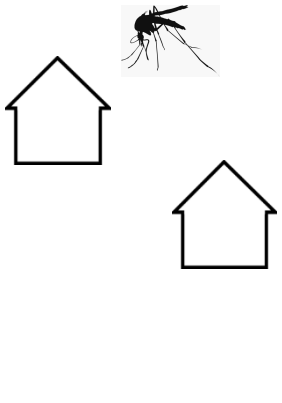
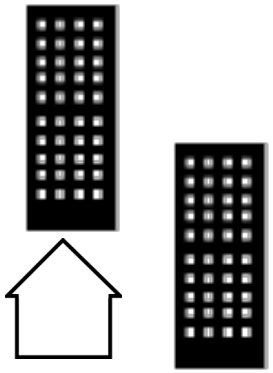
Malaria hotspot



Populated areas near hotspot



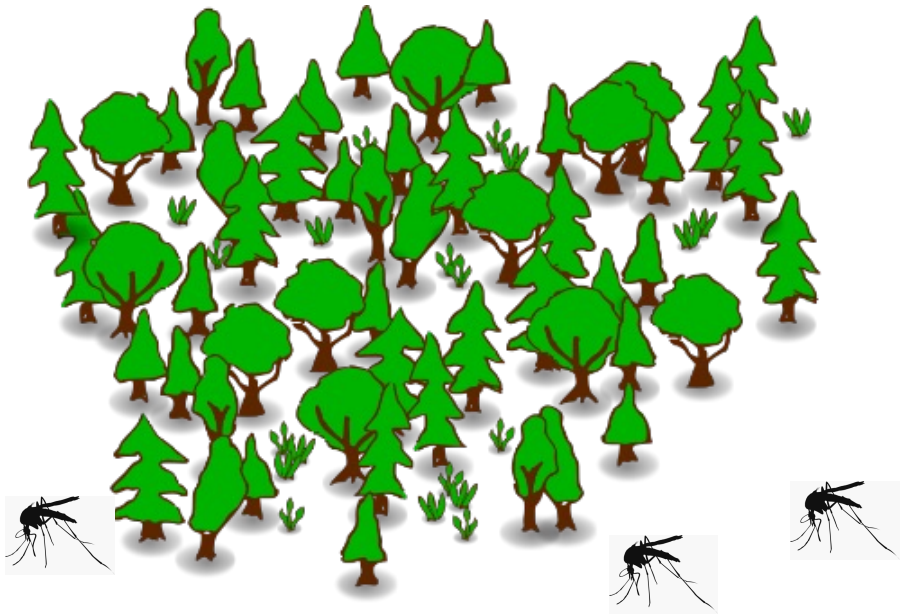
Distant urban areas



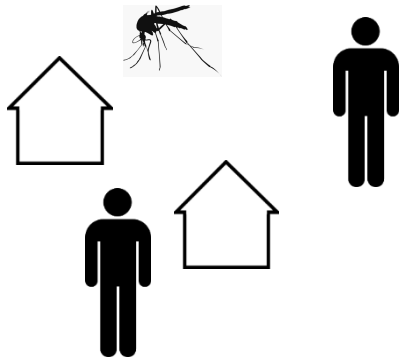
Step 2: **Transit**

Codifying spatial targeting

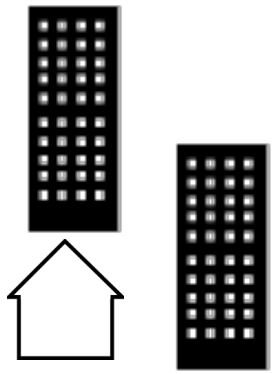
Malaria hotspot



Populated areas near hotspot



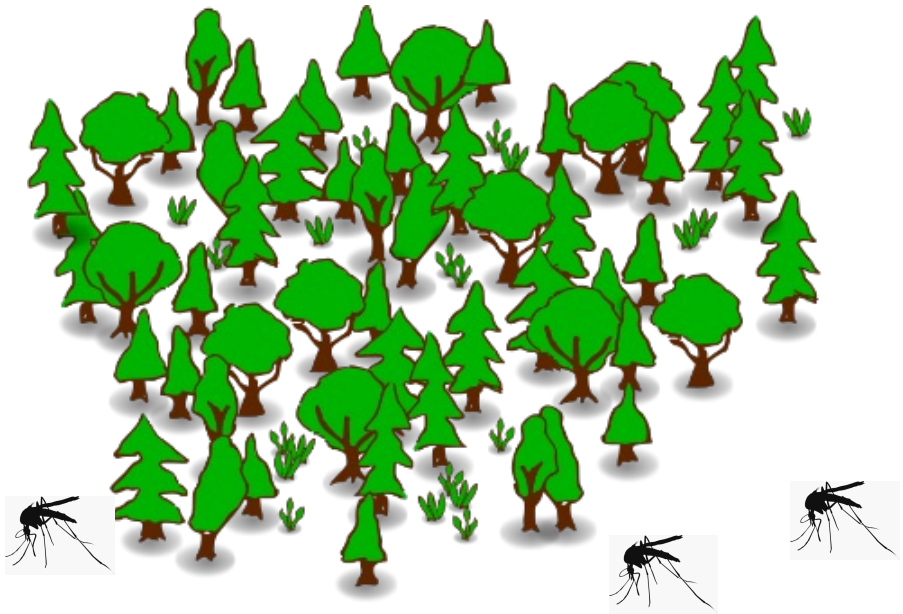
Distant urban areas



Step 3: **Onward transmission**

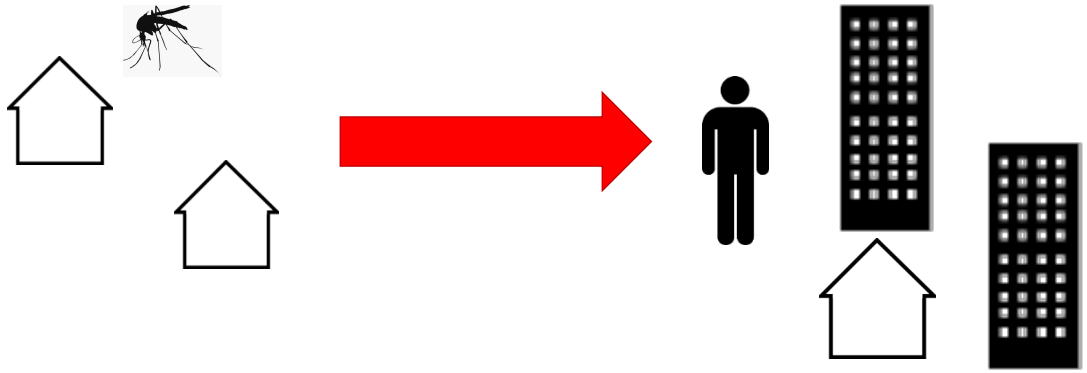
Codifying spatial targeting

Malaria hotspot



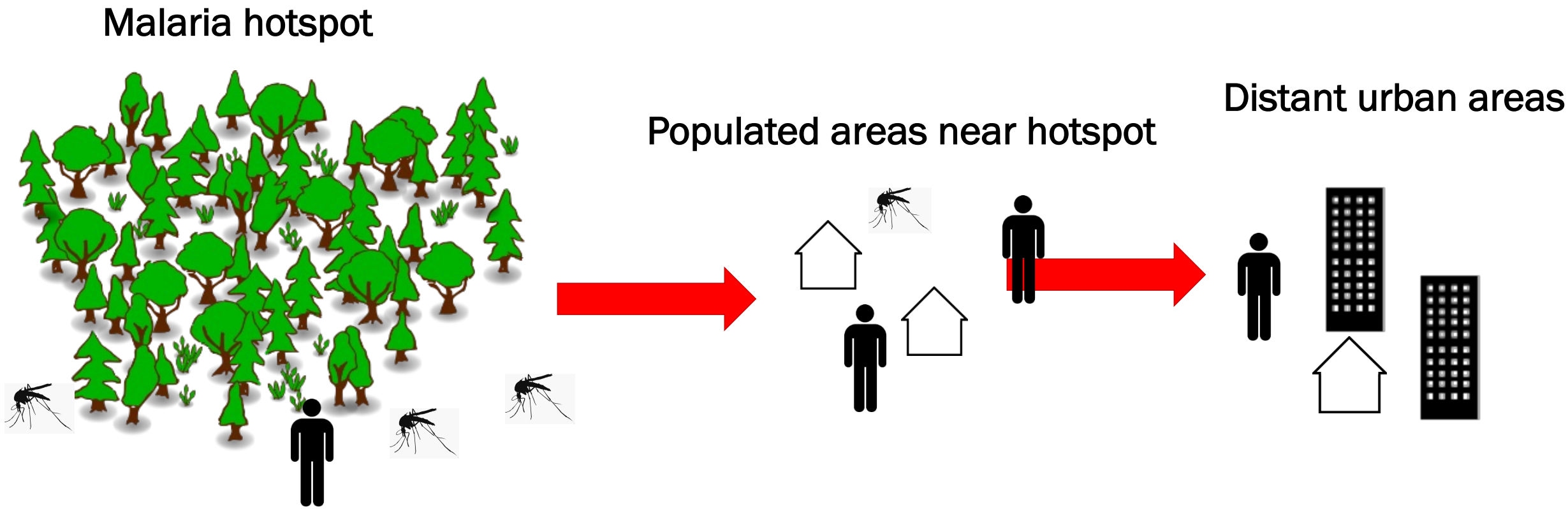
Populated areas near hotspot

Distant urban areas



Onward transmission can continue further afield

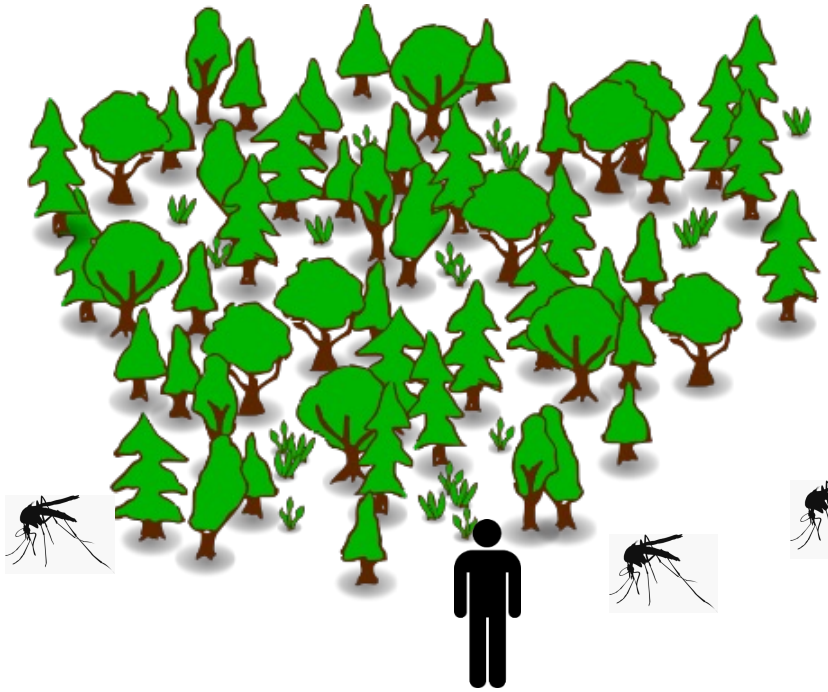
Codifying spatial targeting



In different areas, different types of movement causes this (migration vs. circulatory)

Spatial targeting

Malaria hotspot



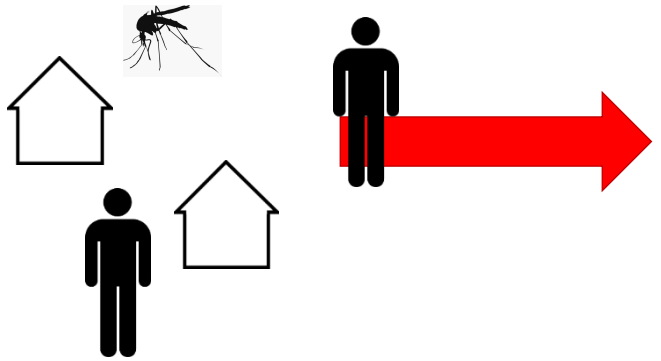
At site of initial infection

Vector control, bednets

During travel

Screening at health facilities

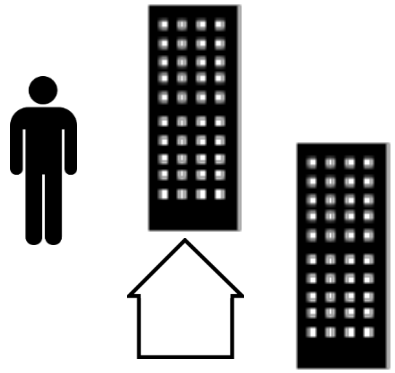
Populated areas near hotspot



In populated areas

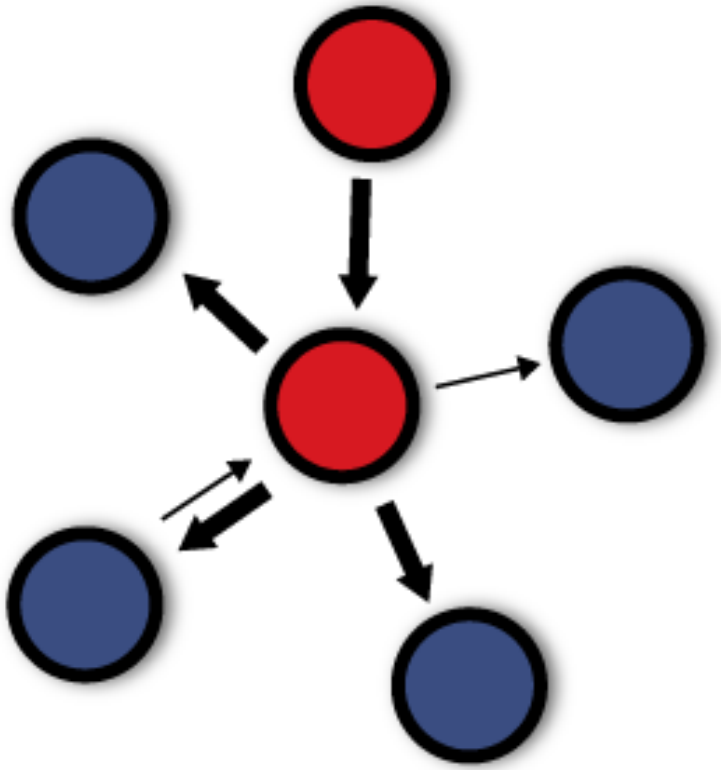
Screening, follow-up case management

Distant urban areas



Factors that drive spatial heterogeneity

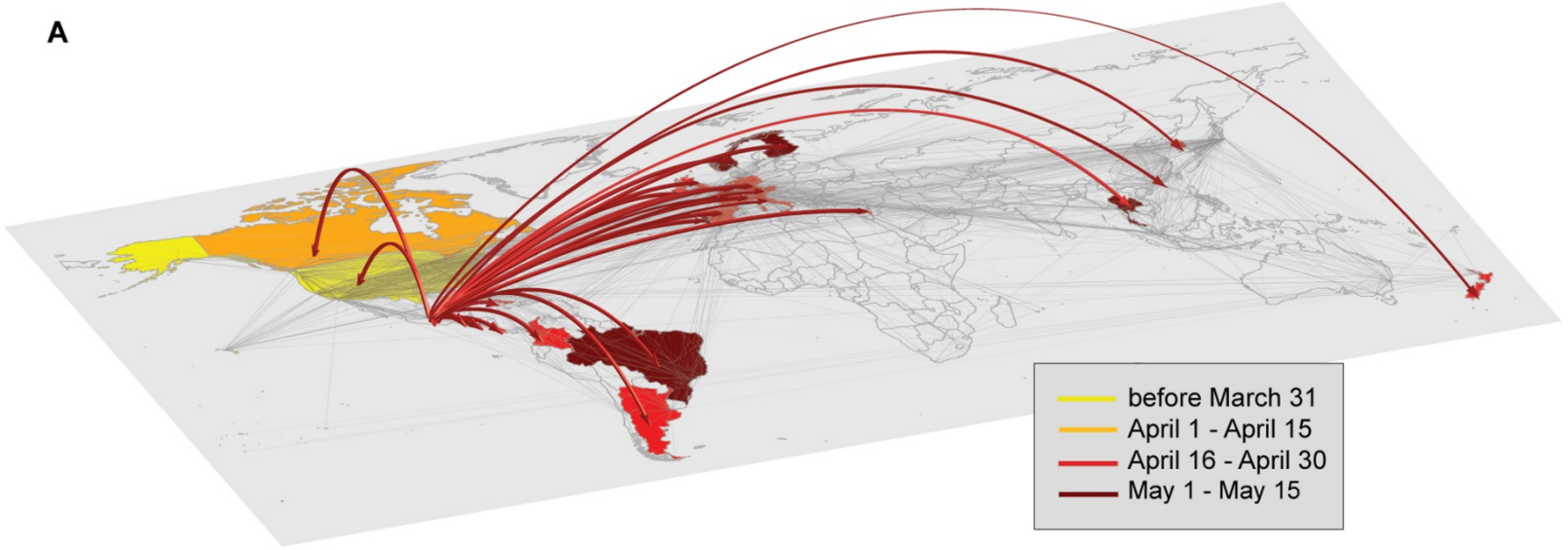
- Movement heterogeneity
- Environmental heterogeneity
- Intervention heterogeneity
- Surveillance heterogeneity



Movement heterogeneity

Host movement

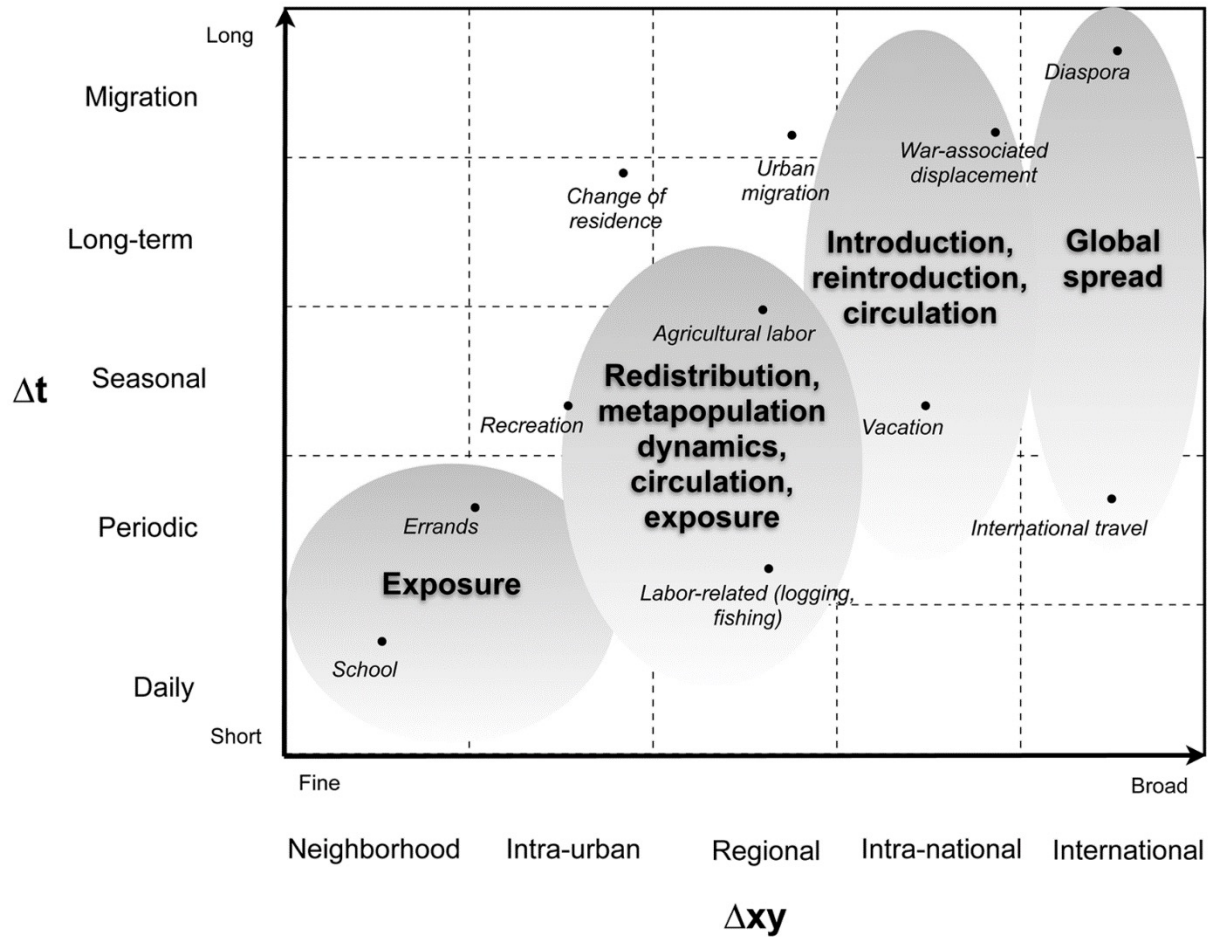
Bajardi et al 2011



- Host movement is fundamental driver of disease spread/emergence
  - **Individual differences**
    - If someone travels more, more contacts
  - **Spatial differences**
    - Some places host large gatherings

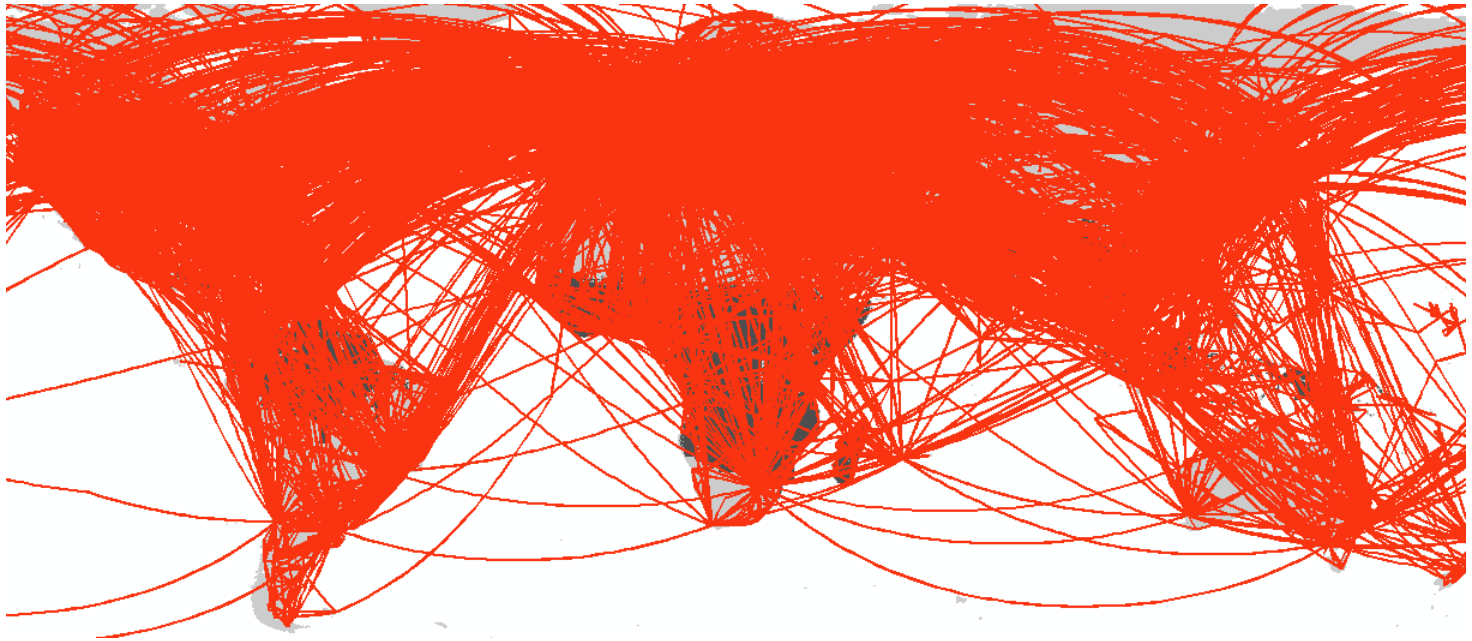
Variance in human mobility

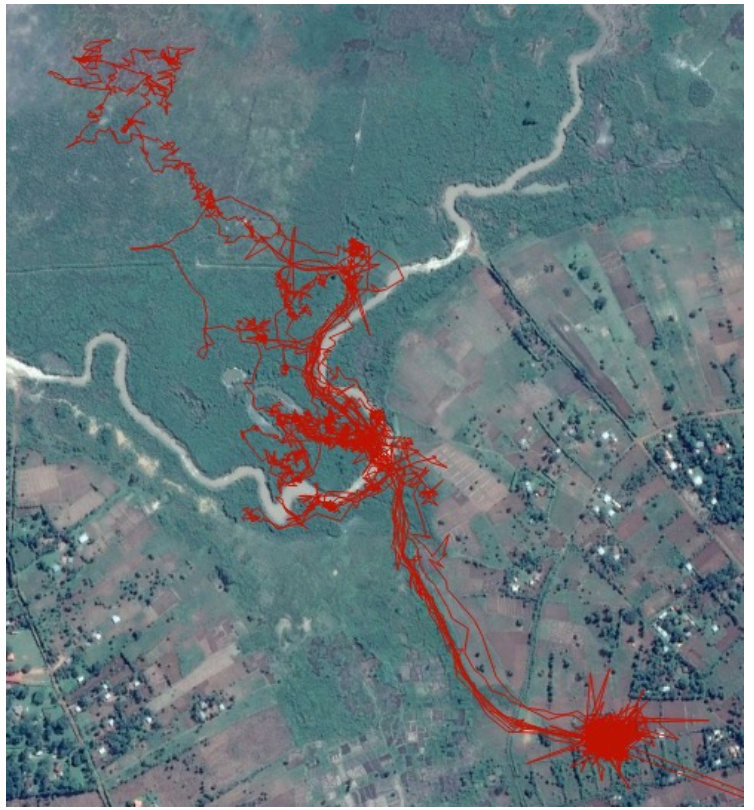
- Humans move for many reasons, across scales
  - Work
  - Leisure
  - School
  - Resources (healthcare, food, etc)
- These types of movements manifest in different ways
  - Long distance, infrequent holiday movement



Stoddard et al 2009. The Role of Human Movement in the Transmission of Vector-Borne Pathogens. PLOS N

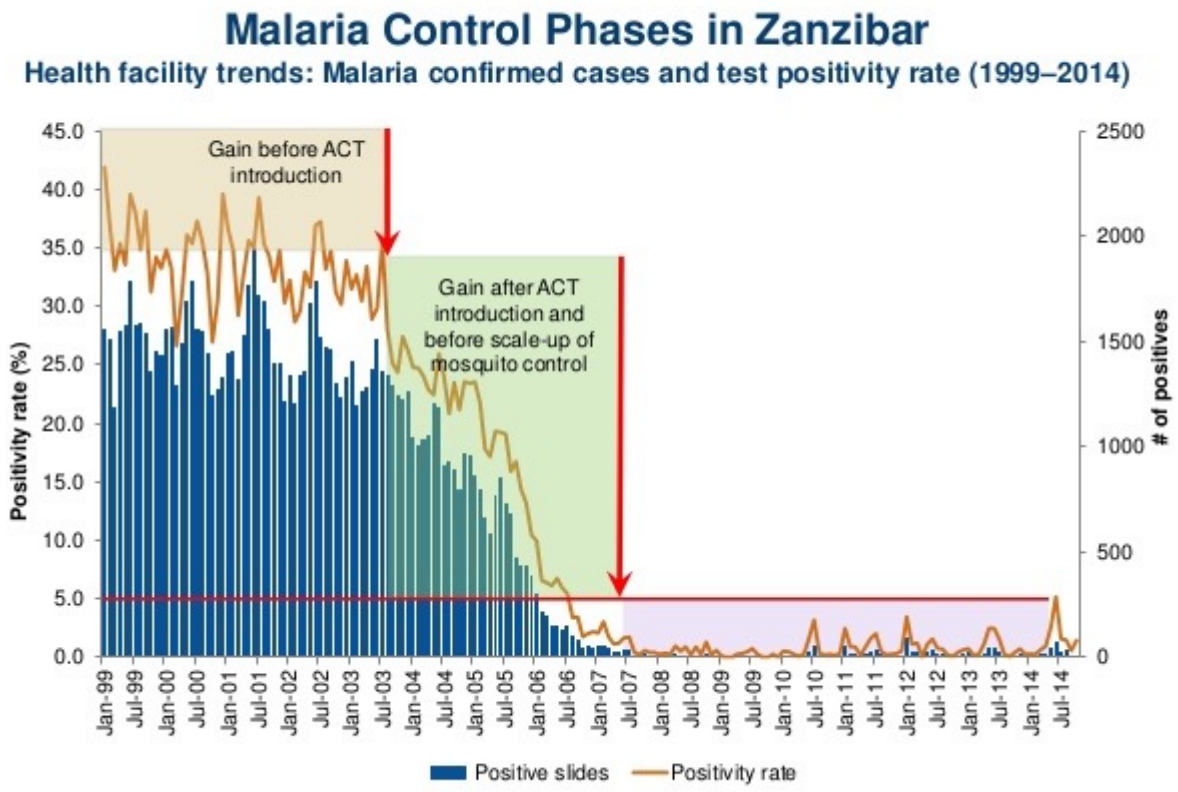
Mobility patterns are important across scales





Human mobility and control

Nearing Malaria Elimination: for the third time



1. Source: Zanzibar Malaria Elimination Program

Human mobility and control

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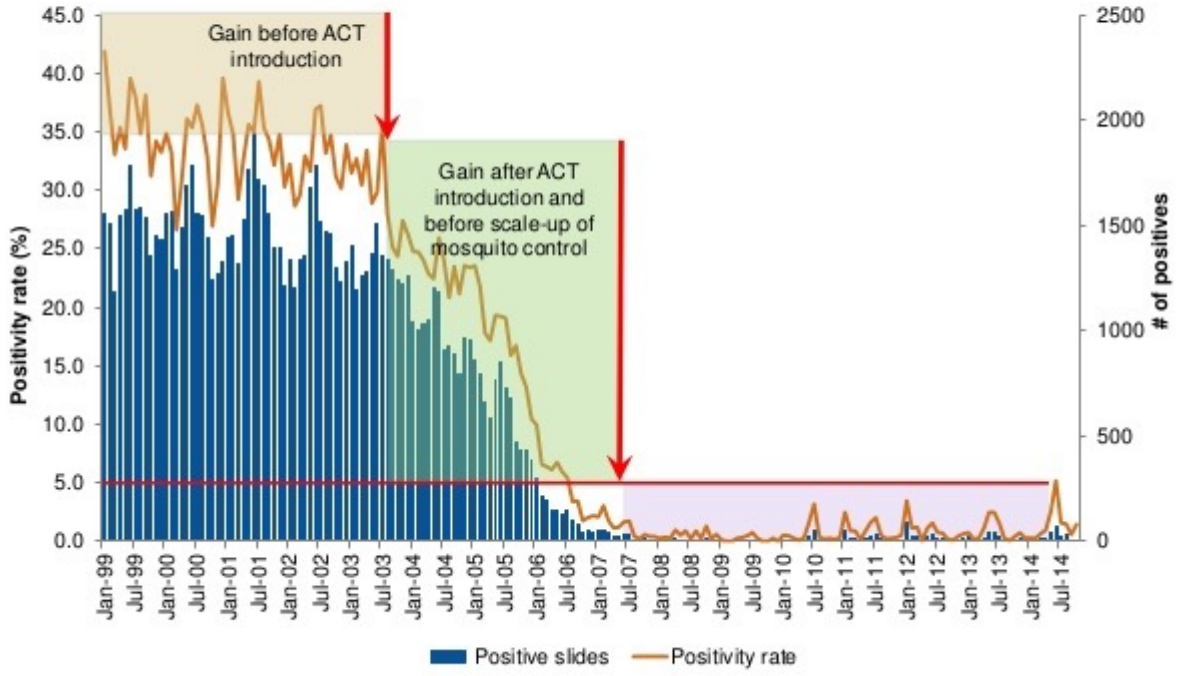
Malaria Control Phases in Zanzibar

Health facility trends: Malaria confirmed cases and test positivity rate (1999–2014)



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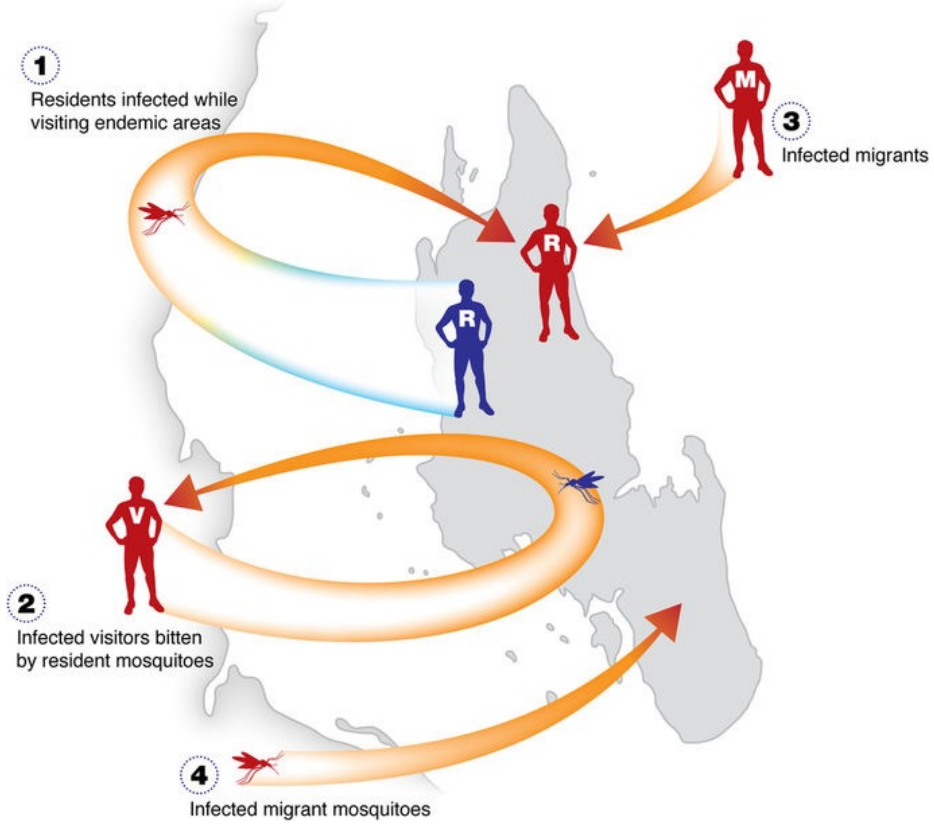

Mobility heterogeneity and models

$$S_i(t+1) = S_i(t) - \beta_i \frac{S_i(t)I_i(t)}{N_i(t)} + \sum_j \boldsymbol{m_{ji}\, S_j(t)} - \sum_j \boldsymbol{m_{ij}\, S_i(t)}$$

$$I_i(t+1) = I_i(t) + \beta_i \frac{S_i(t)I_i(t)}{N_i(t)} - \gamma_i I_i(t) + \sum_j \boldsymbol{m_{ji}\, I_j(t)} - \sum_j \boldsymbol{m_{ij}\, I_i(t)}$$

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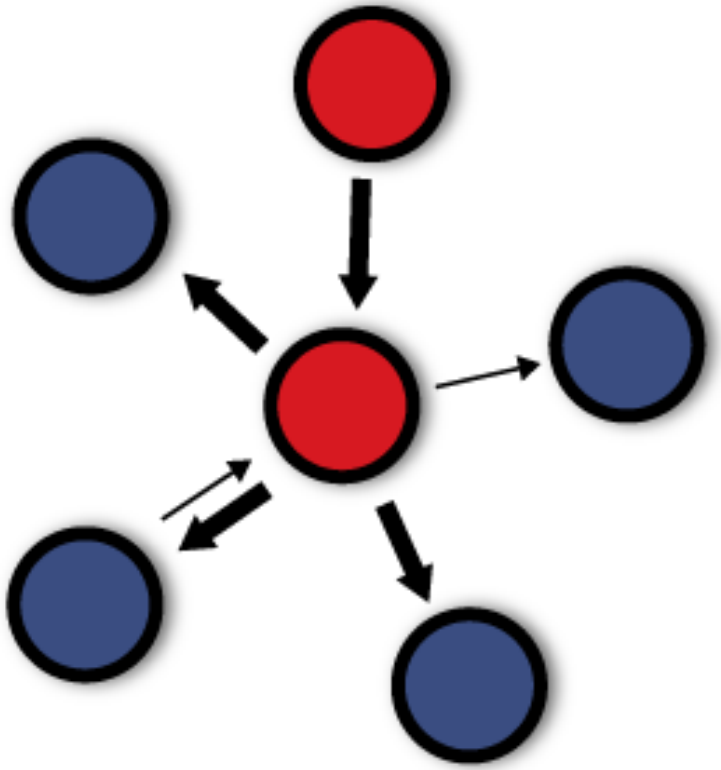
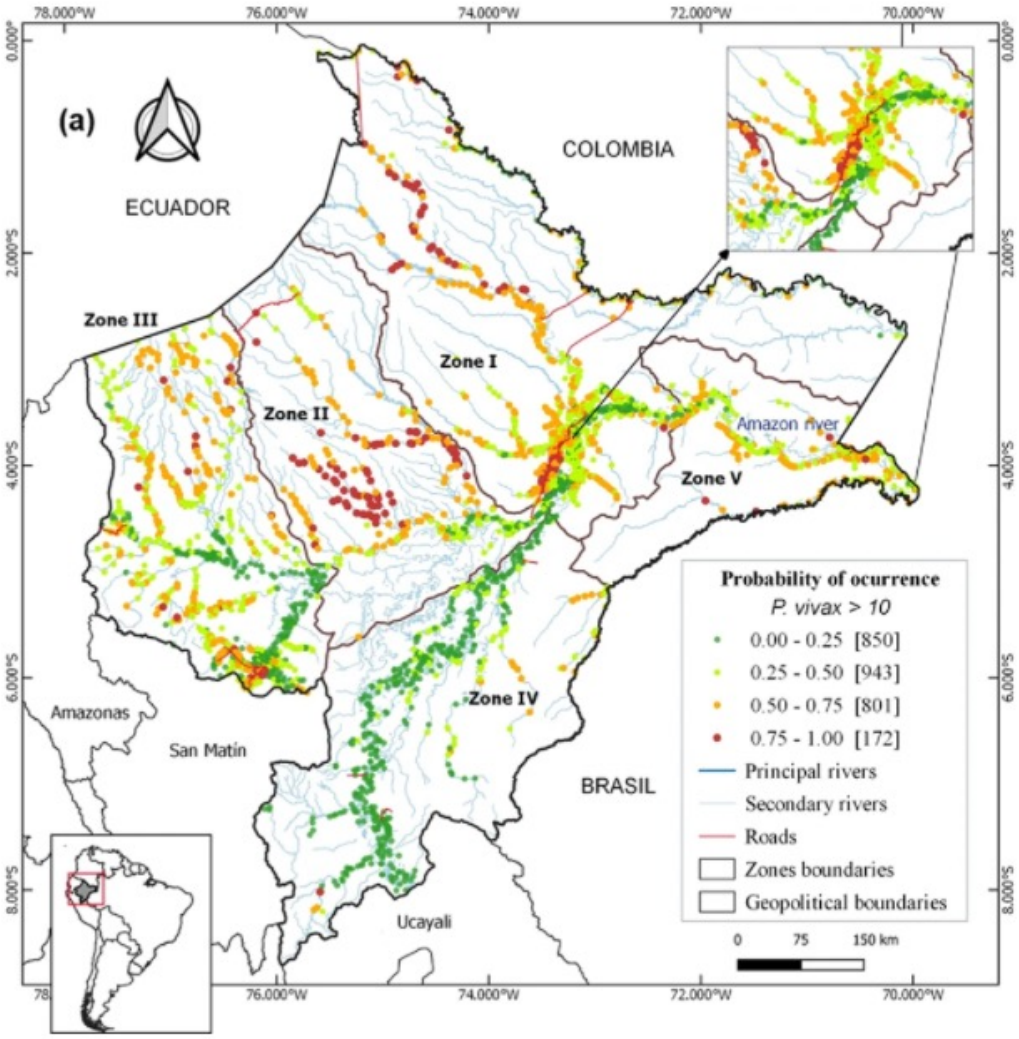
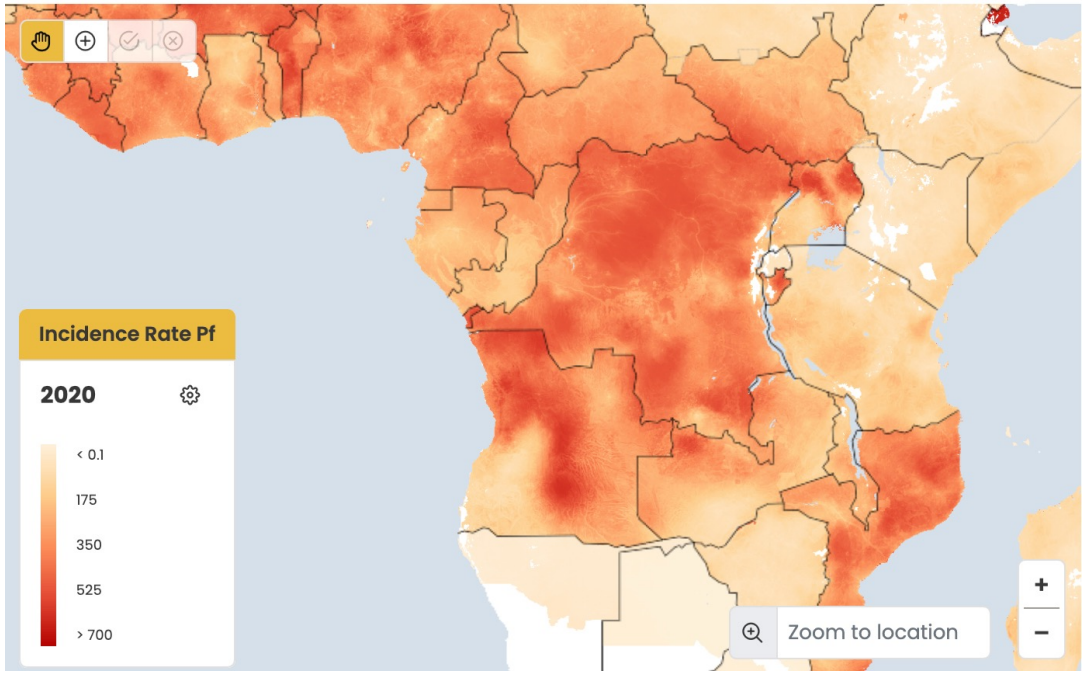
- Environmental heterogeneity
- Intervention heterogeneity
- Surveillance heterogeneity



Environmental heterogeneity

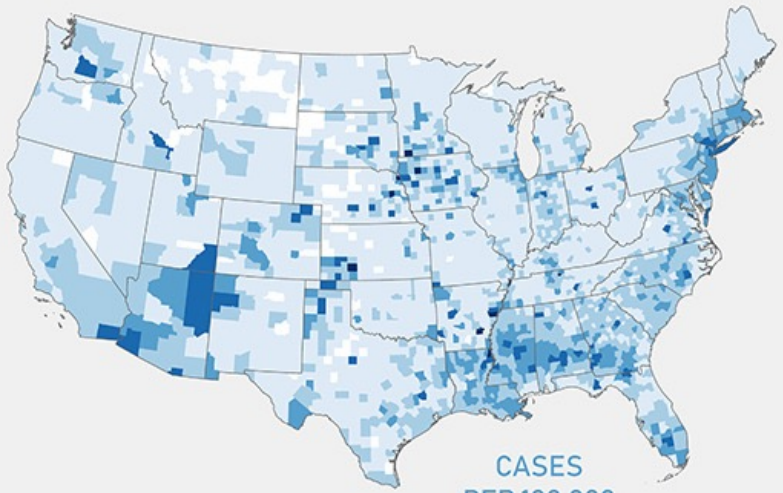
Environmental heterogeneity



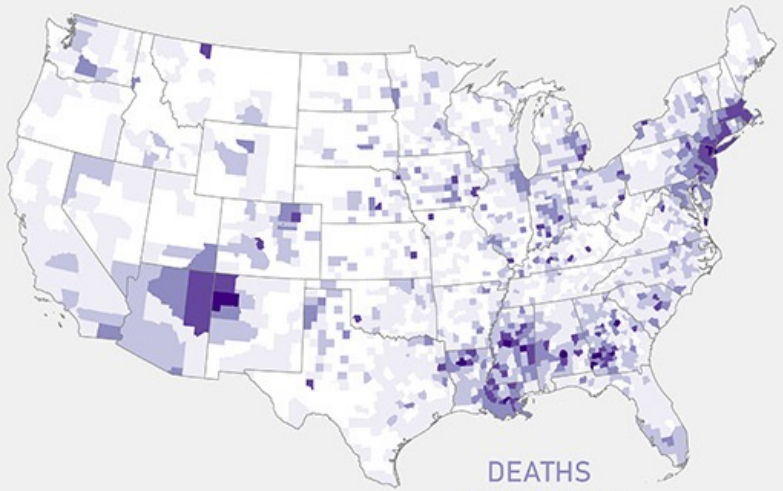


Analyzing the spatial determinants of local Covid-19 transmission in the United States

CUMULATIVE COVID-19 CASE AND DEATH DATA (JANUARY THROUGH JUNE)

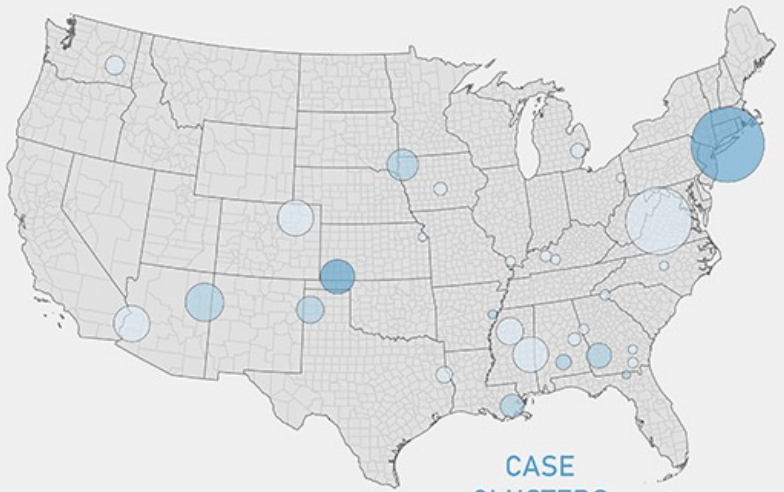


CASES PER 100,000

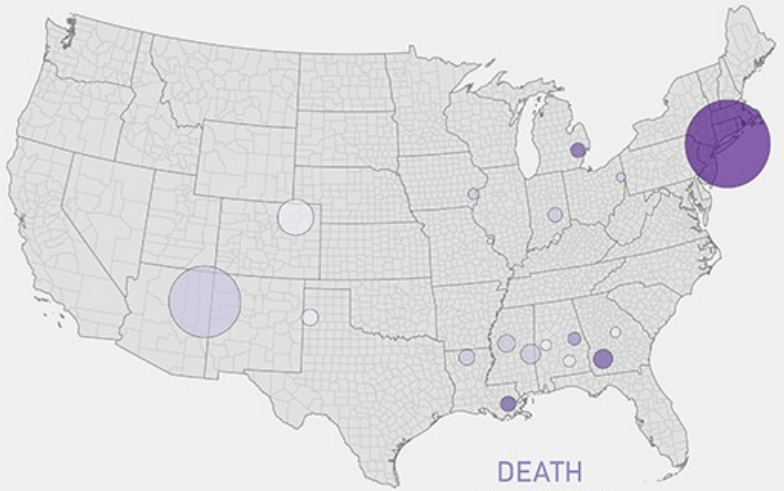


DEATHS PER 100,000

CLUSTER ANALYSIS REVEALS CLUSTERS IN THE NORTHEAST, SOUTHEAST, AND SOUTHWEST



CASE CLUSTERS



DEATH CLUSTERS

REGRESSION ANALYSIS REVEALS SIX SIGNIFICANT PREDICTORS OF COVID-19 CASES AND DEATHS

% SPEAKING ENGLISH LESS THAN WELL

% EMPLOYED IN PRODUCTION OCCUPATIONS

RURAL-URBAN COMMUTING AREA CODE

% BLACK OR AFRICAN AMERICAN POPULATION

% 65 YEARS OLD AND OLDER

% WITH A DISABILITY

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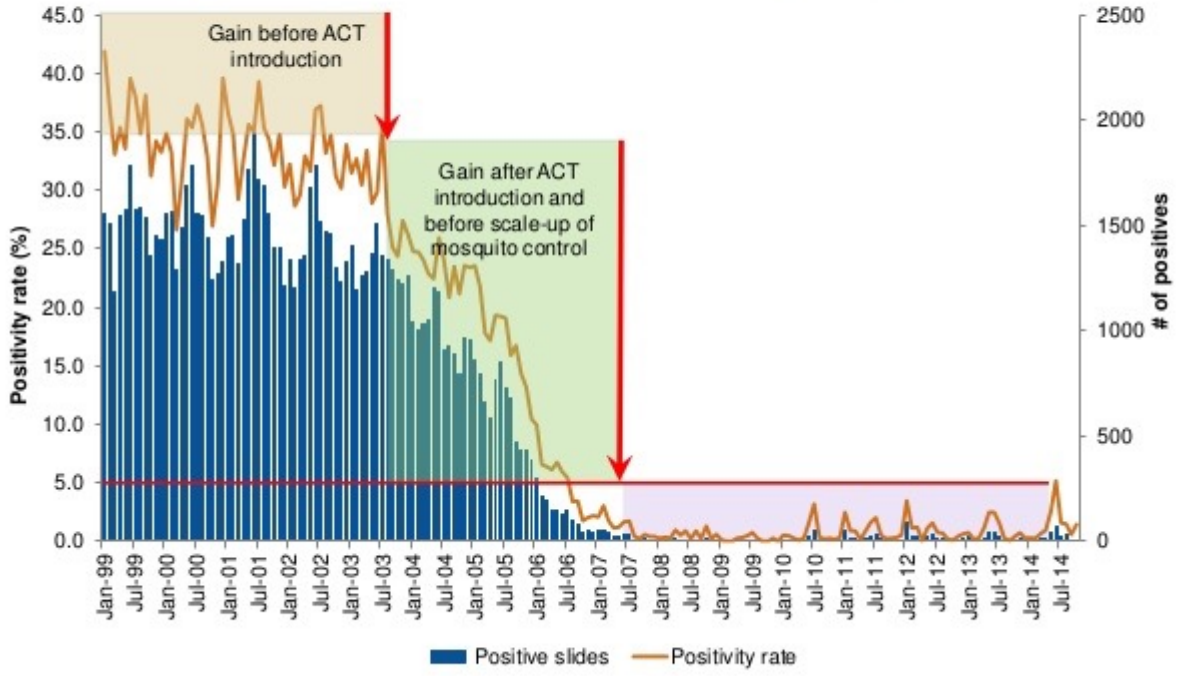

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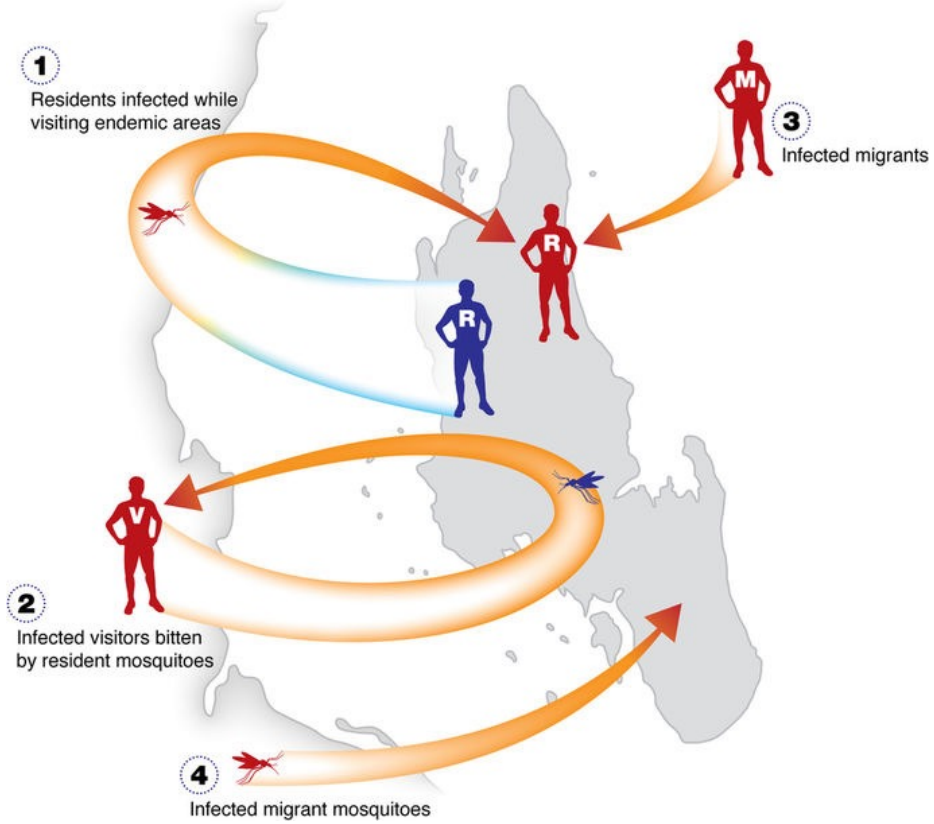

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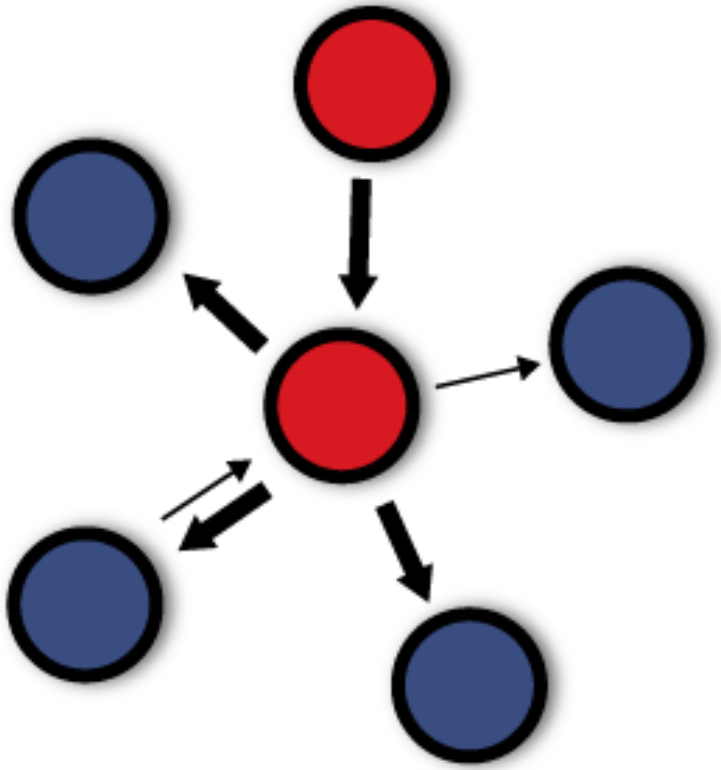
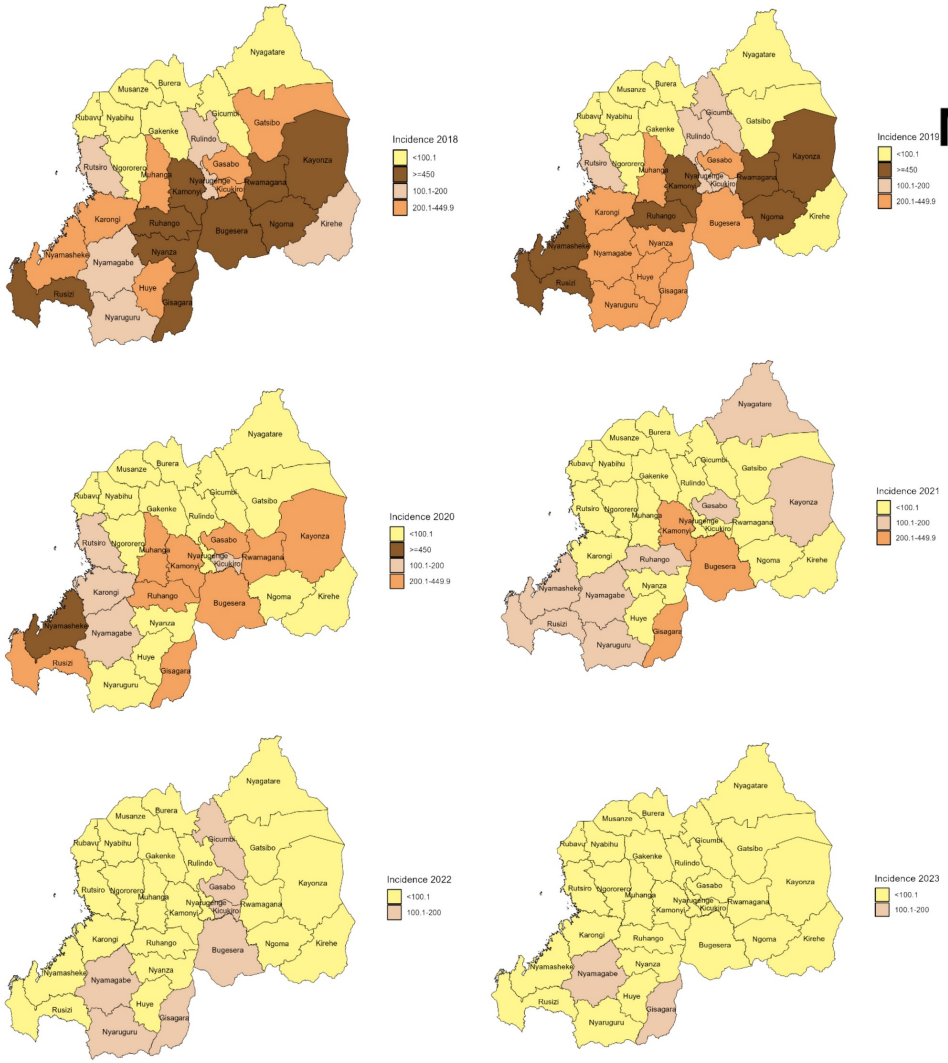
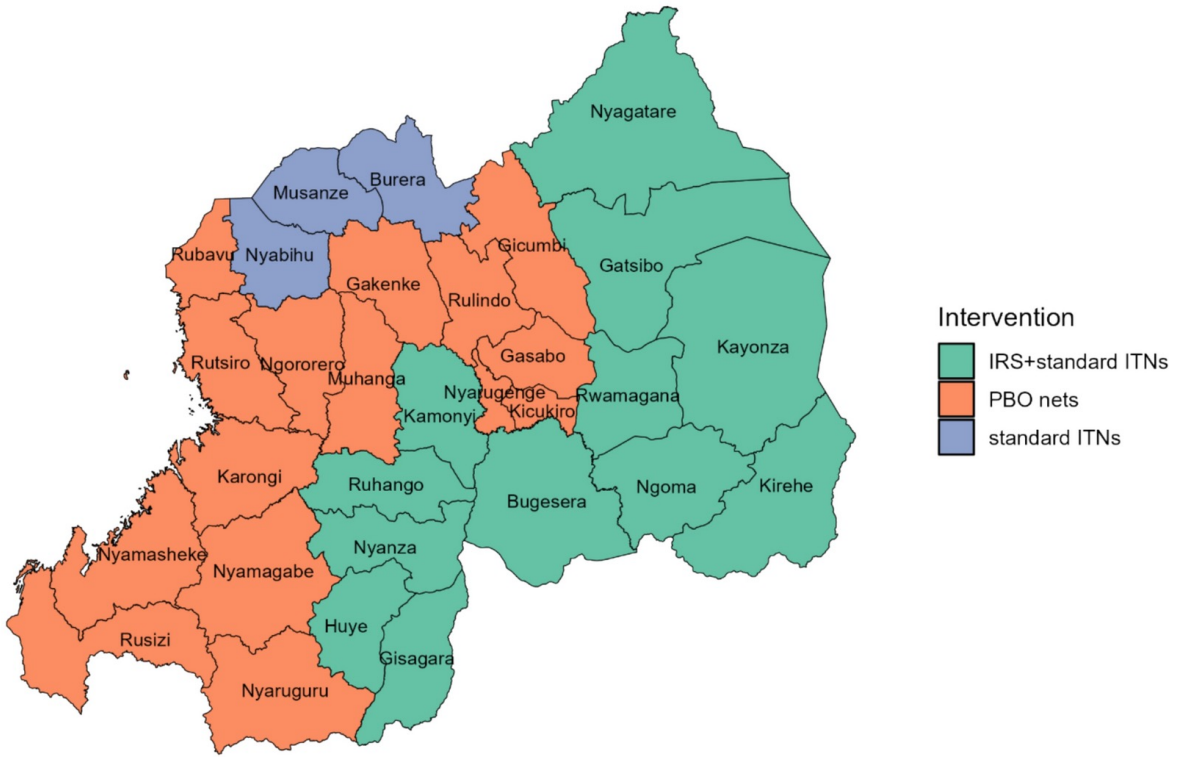
- Environmental heterogeneity
  - Influences what areas are “sources” and “sinks”
  - Can change over time!
- Intervention heterogeneity
- Surveillance heterogeneity



Intervention heterogeneity

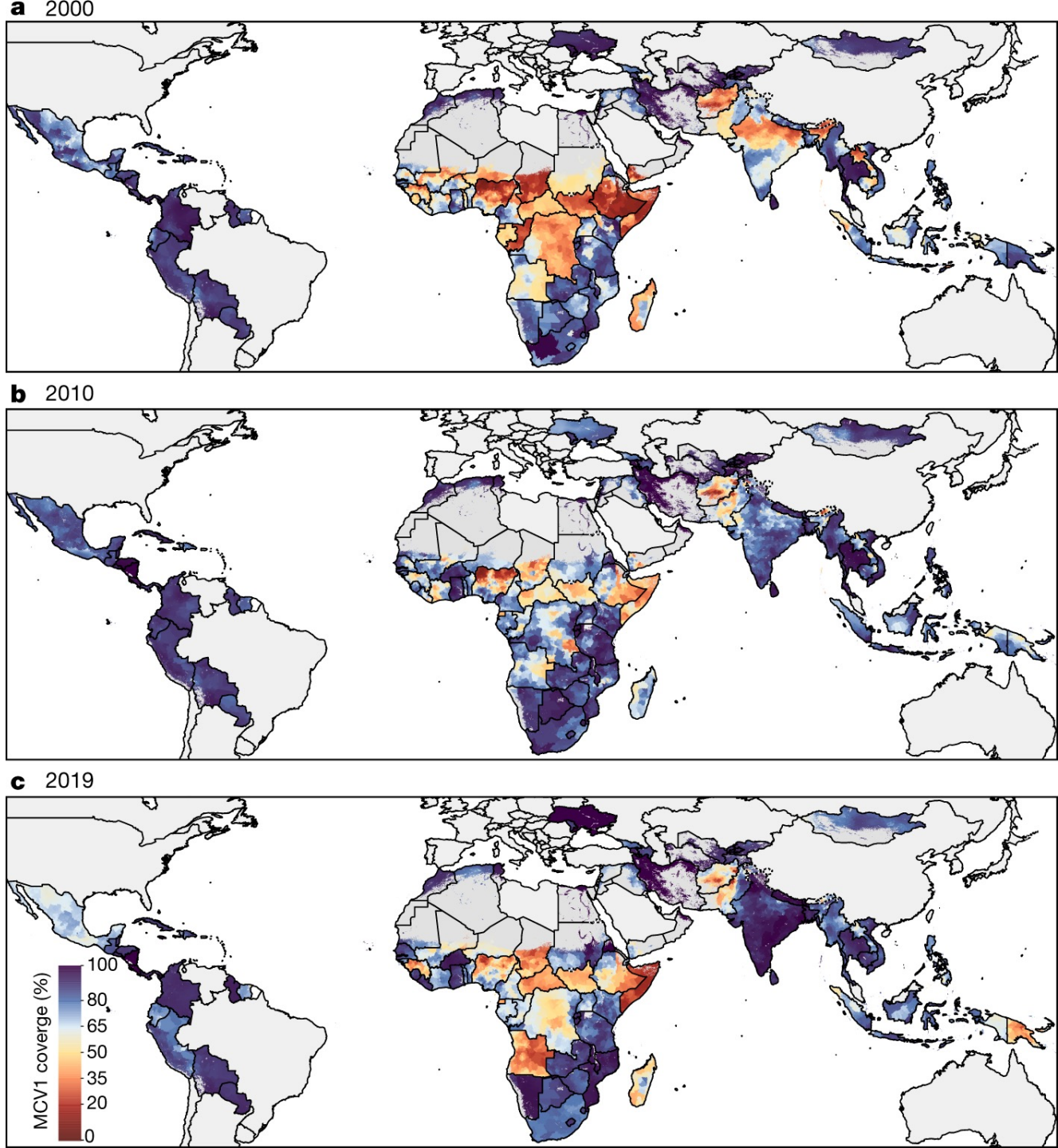
Intervention heterogeneity





https://link.springer.com/article/10.1186/s12936-025-05278-w/figures/3

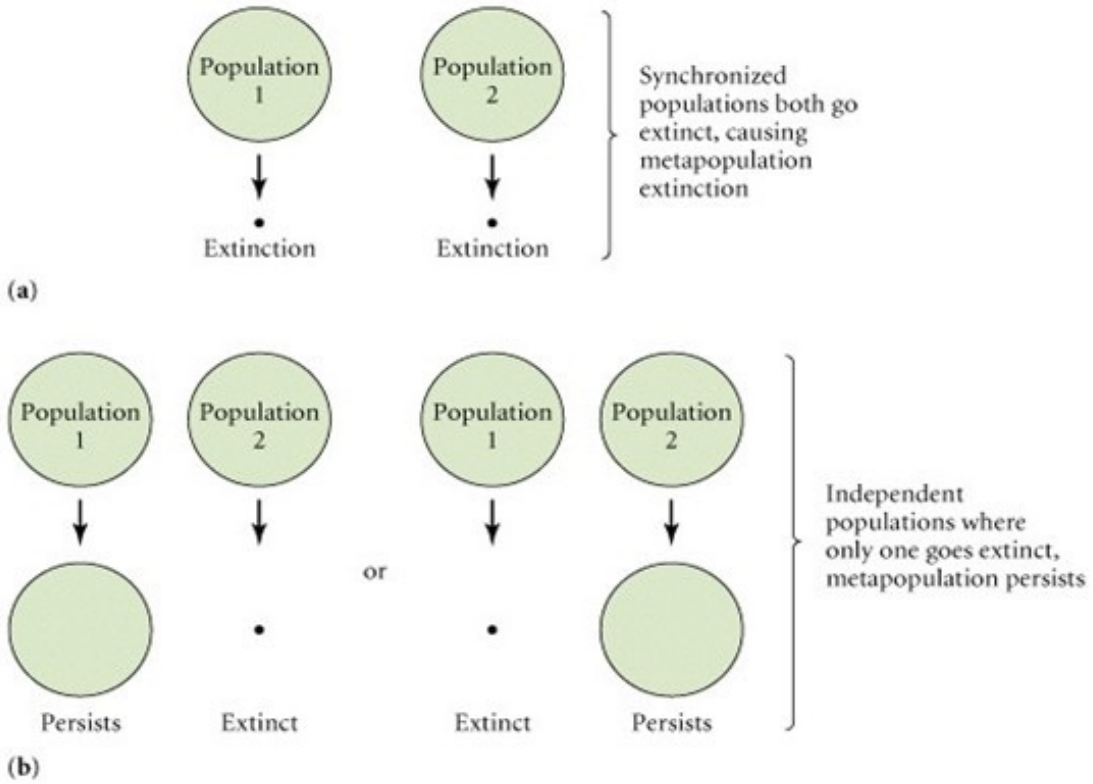
Measles vaccination coverage



https://www.nature.com/articles/s41586-020-03043-4

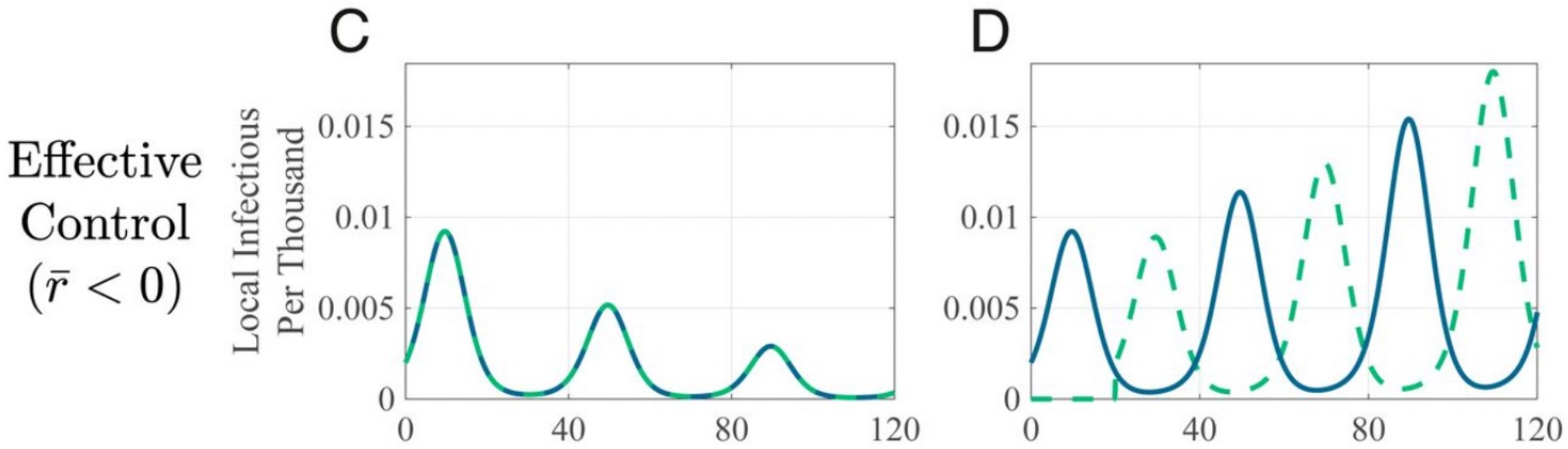
Synchrony/asynchrony

- If populations are synchronized, more likely that extinctions will happen at the same time
  - High synchrony: global extinction possible
  - Low synchrony: Regional persistence



Timing control makes a big difference

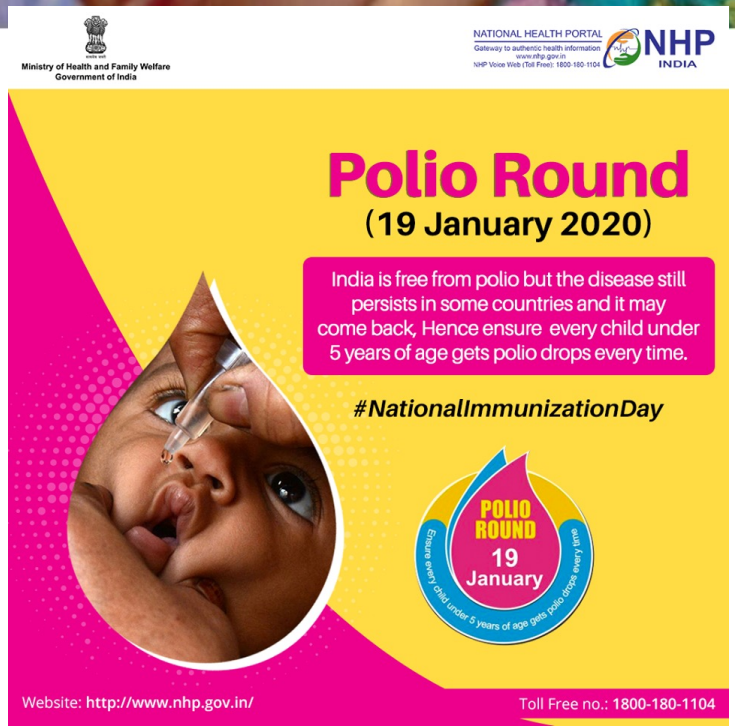
- Different interventions can induce “synchrony”/”asynchrony”
  - Everybody under self-isolation at the same time
- Seasonality in behavior can, too



https://www.pnas.org/content/117/48/30104

Pulsed vaccination

- Attempt to synchronize effect of vaccination across subpopulations
- Strong synchronized reductions hopefully drop things to zero
- Common for polio



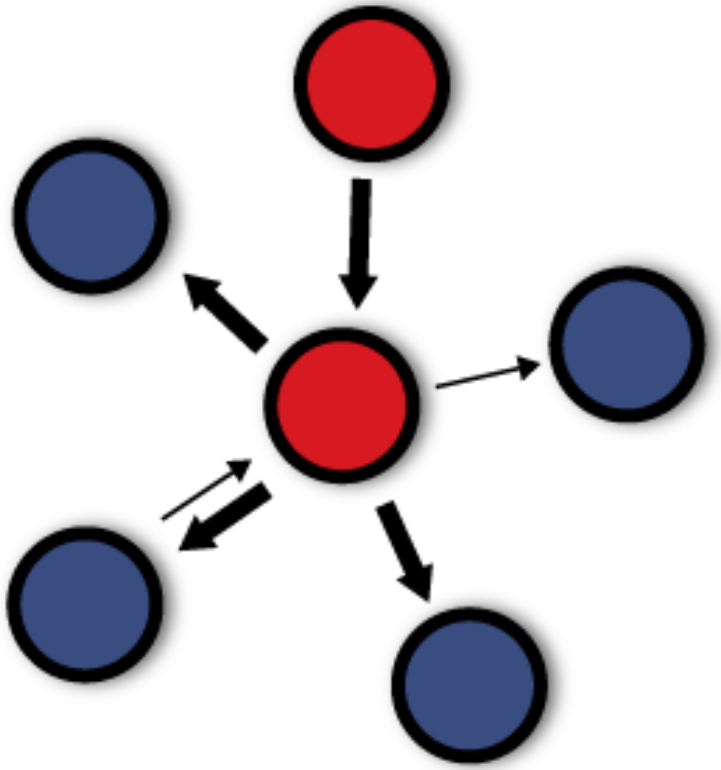
$$S_i(t+1) = S_i(t) - \boldsymbol{\beta_i} \frac{S_i(t) I_i(t)}{N_i(t)} + \sum_j m_{ij}\, S_j(t) - \sum_j m_{ji}\, S_i(t)$$

$$I_i(t+1) = I_i(t) + \boldsymbol{\beta_i} \frac{S_i(t) I_i(t)}{N_i(t)} - \boldsymbol{\gamma_i} I_i(t) + \sum_j m_{ij}\, I_j(t) - \sum_j m_{ji}\, I_i(t)$$

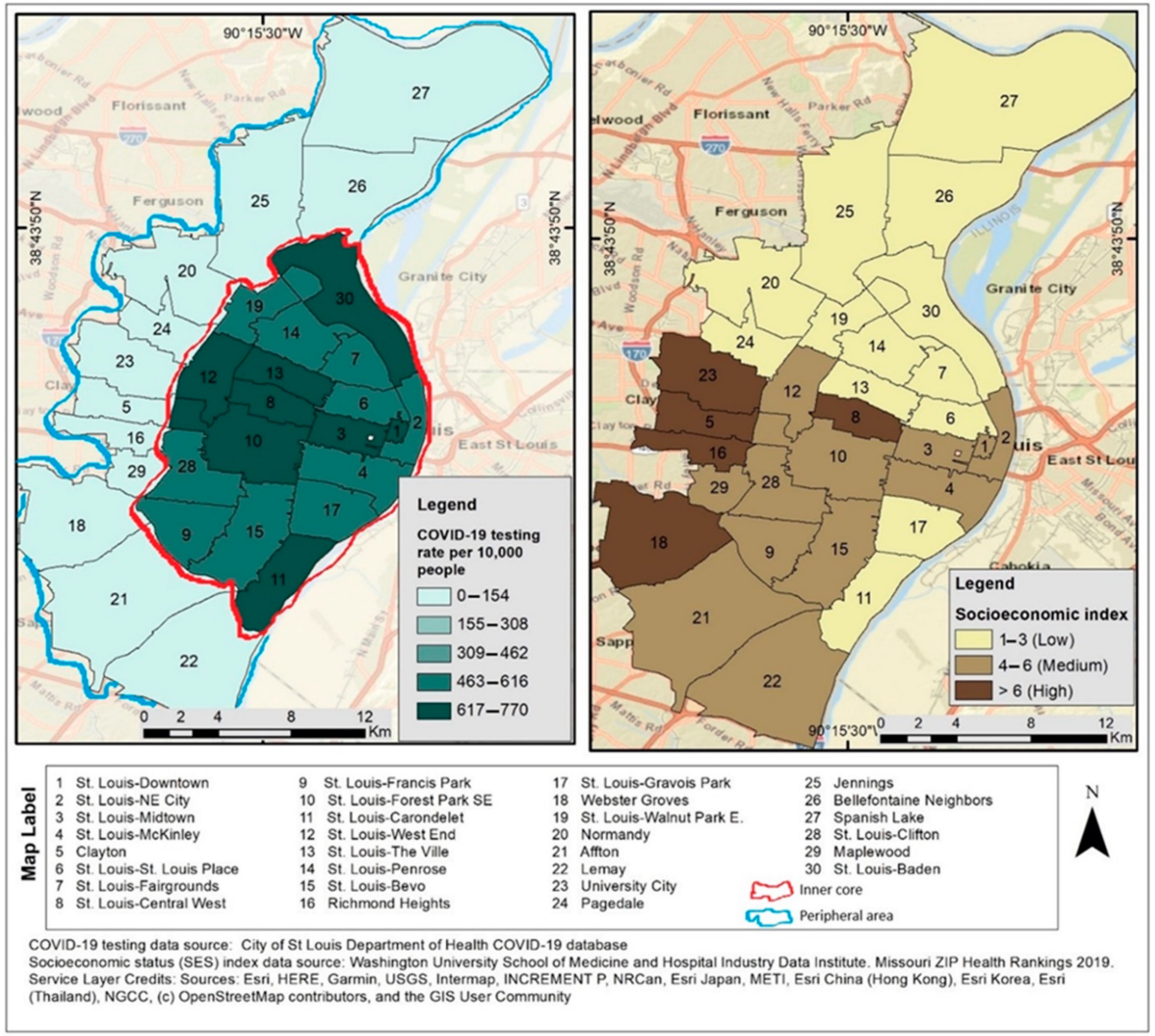
$$R_i(t+1) = R_i(t) + \boldsymbol{\gamma_i} I_i(t) + \sum_j m_{ij}\, R_j(t) - \sum_j m_{ji}\, R_i(t)$$

Factors that drive spatial heterogeneity

- Movement heterogeneity
  - Impacts spread of disease, flows out of “sources” and into “sinks”
- Environmental heterogeneity
  - Influences what areas are “sources” and “sinks”
  - Can change over time!
- Intervention heterogeneity
  - Affects sources/sinks
  - Can be used to help synchronize outbreaks
- Surveillance heterogeneity

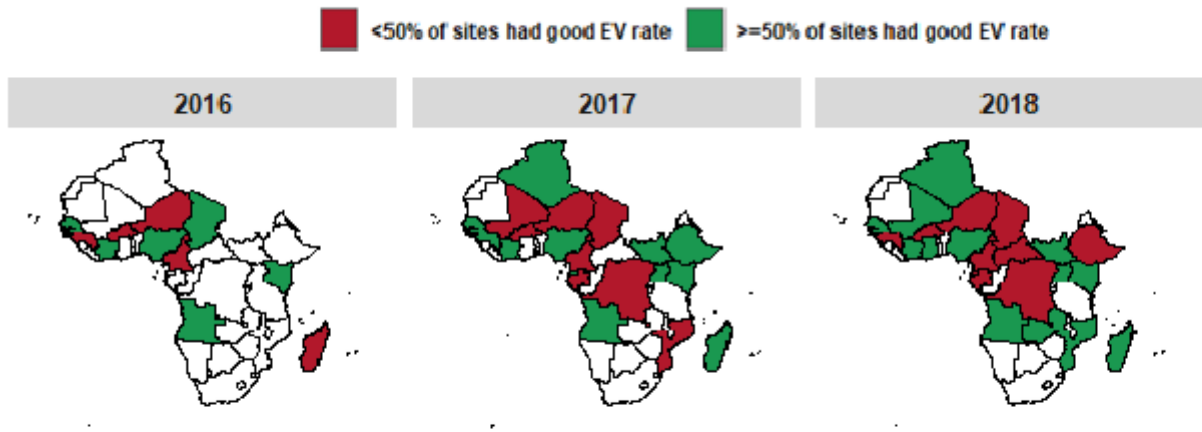


Surveillance heterogeneity



https://www.mdpi.com/2673-8112/4/9/103

Fig. 9. Overall enterovirus detection among environmental surveillance sites by country – African Region, 2016–2018



Enterovirus (EV) = WPV, VDPV, SL, or NPEV; WPV: Wild poliovirus; VDPV: Vaccine-derived poliovirus; SL: Sabin-like virus; NPEV: Non-polio enterovirus

Source: WHO.

$$S_i(t+1) = S_i(t) - \beta_i \frac{S_i(t)\boldsymbol{I_i(t)}}{N_i(t)} + \sum_j m_{ij}\, S_j(t) - \sum_j m_{ji}\, S_i(t)$$

$$I_i(t+1) = \boldsymbol{I_i(t)} + \beta_{\boldsymbol{i}} \frac{S_i(t)\boldsymbol{I_i(t)}}{N_i(t)} - \gamma_i \boldsymbol{I_i(t)} + \sum_j m_{ij}\, I_j(t) - \sum_j m_{ji}\, I_i(t)$$

$$R_i(t+1) = R_i(t) + \gamma_i \boldsymbol{I_i(t)} + \sum_j m_{ij}\, R_j(t) - \sum_j m_{ji}\, R_i(t)$$

Factors that drive spatial heterogeneity

- Movement heterogeneity
  - Impacts spread of disease, flows out of “sources” and into “sinks”
- Environmental heterogeneity
  - Influences what areas are “sources” and “sinks”
  - Can change over time!
- Intervention heterogeneity
  - Affects sources/sinks
  - Can be used to help synchronize outbreaks
- Surveillance heterogeneity
  - Influences whether outbreaks are accurately detected